File D 00.12/122/No. 3533

AIR CORPS TECHNICAL REPORT NO. 3533

AIR CORPS INFORMATION CIRCULAR

PUBLISHED BY THE CHIEF OF THE AIR CORPS, WASHINGTON, D. C.

Vol. VII — February 23, 1932 — No. 664

VIBRATION CHARACTERISTICS OF AIRCRAFT ENGINE CRANK SHAFTS

▽

(POWER PLANT BRANCH REPORT)

UNITED STATES
GOVERNMENT PRINTING OFFICE
WASHINGTON : 1932

INDEX

VIBRATION CHARACTERISTICS OF AIRCRAFT ENGINE CRANKSHAFTS

(Prepared by F. L. Prescott, Matériel Division, Air Corps, Wright Field, Dayton, Ohio, October 2, 1931)

OBJECT

These tests were made to develop a satisfactory instrument for recording torsional vibrations and to make possible calculation of frequency of vibration from drawings.

SUMMARY

A very satisfactory instrument has been devised for recording torsional vibration. Many tests have been made and vibration records taken on engines of all makes and types. These records are presented and discussed. The method of calculating frequency of vibration is presented in its simplest form, and results of calculations with corresponding observed frequencies are tabulated.

CONCLUSIONS

Calculation of critical ranges due to torsional vibration can be made with sufficient accuracy to make the study of great value in crankshaft design.

INTRODUCTION

It has long been realized by designers of internal combustion engines that calculation of stresses in crankshafts was incomplete without an investigation of the torsional vibration characteristics of the engine. Marine designers have been compelled to take this phase of design into account because of the nature of the drives used and the extremely favorable conditions for large amplitude torsional vibrations. It is only because of the shorter drives and the comparatively stiffer crankshafts used in aircraft engines that the torsional periods have not forced themselves on the designers of such engines at an earlier stage. There has also been a feeling among those actively engaged in such work that calculation of torsional periods was too involved and complex and too little was known of the phenomenon to make it worth while to take account of it. As a result, engines are built and run regardless of critical periods and when a "rough spot" is observed in the operating range, the engine speed is varied to avoid running at that particular point. An example in point is the Liberty "12" aircraft engine. Calculation of crank shaft stresses does not reveal any marked weakness, and yet there are speeds within the operating range where crank shaft failures occur. It has been found by numerous tests that there are three critical speeds at or near which torsional vibration is encountered. These speeds, as will be pointed out in connection with test records, are 1,000 r. p. m., 1,333 r. p. m., and 1,715 r. p. m. with a frequency of 100 vibrations per second. Two of these periods, most unfortunately, occur at speeds which are encountered in operation. The normal rated speed of the Liberty "12" is 1,700 r. p. m., and when fitted with a propeller for supercharging, but taking off without supercharge, the speed is pulled down to 1,350 r. p. m., or 1,400 r. p. m. A consideration of this design from the torsional vibration standpoint would have indicated the desirability of removing these periods to a little-used part of the speed range. Because of the smoothness of operation of the Liberty "12" at 1,450 r. p. m. to 1,550 r. p. m., the pilot prefers to use this speed range at the expense of the output. The Packard 2A–2500 12-cylinder Vee engine has a critical period of vibration at 1,800 r. p. m., its rated speed. The roughness of this engine, which was partially due to poor distribution, caused numerous failures of pipes, accessories, and water joints. To remedy this serious effect, an Allison spring coupling was incorporated in the geared models, which seems to have the effect of making all speeds about equally bad in respect to torsional vibration. See Figures 12 and 14. The coupling did, however, reduce the impact load on the teeth of the reduction gearing.

It would appear to be ideal to so design a crankshaft that no torsional periods would occur within its operating range. This is readily done in the case of the radial engine because of the short, stiff crank shaft used. In the case of the 4 or 6 cylinder line engine and the 8-cylinder V, it is still possible to avoid the primary critical period. The 4 and 6 cylinder line engines, however, will have torsional periods at one-half the primary critical speed by reason of the double frequency inertia torque which is characteristic of these engines.

The inertia torque characteristics of a number of engine types have been computed and published in the Automobile Engineer, September, 1929. In the curves, the fact is brought out that the torque due to inertia is, in the 4 and 6 cylinder arrangements, greatest in the propeller and crank cheek and main journal. In the 12-cylinder V, the stress due to this factor is greater in the journal between cylinders 5 and 6. This would seem to explain why the Liberty crank shaft almost always breaks at one of the cheeks between 5 and 6 instead of the last cheek. In the case of the radial engine, inertia torque is a negligible factor.

In case of the 8-cylinder and 12-cylinder V, the frequency of the inertia torque is equal to that of the power impulses. This frequency is too high in the 12-cylinder V to permit of so designing the crank shaft as to eliminate torsional vibration, even up to a speed of 2,000 r. p. m., since the equivalent flywheel effect of the necessary reciprocating and rotating parts combined with that of the crank shaft itself would make necessary a stiffness which can not be incorporated in a practical design. It therefore appears that the best that can be done is to place the rated speed just below one of the critical ranges. This allows a considerable throttle range before the next lower period is reached. The reduced torque impulses at this latter period are not sufficiently serious to cause dangerous torsional vibration.

The study of torsional vibration by the Power Plant Branch, Matériel Division of the United States Army Air Corps, began at McCook Field prior to July, 1927. It was not, however, seriously undertaken until the summer months of that year. The project was carried through the summers of 1927 and 1928 and from June, 1929, until the present. It is felt that there is still much to be learned, in spite of the mass of data already taken.

TEST MATERIAL AND APPARATUS

The question of an instrument suitable for recording the vibrations actually encountered proved to be a serious one. Many types were considered and discarded as impractical for various reasons. The electromagnetic type of instrument gives a beautiful record of the relative velocity of the crank shaft and some uniformly rotating reference member. This record must be integrated in order to show the displacement record, and of course does not give the amplitude of vibration. In the case of a vibration of harmonic form, the velocity record also gives the frequency, but the rather complicated displacement curves observed are not readily studied by any record other than the displacement-time record. It was, therefore, decided that the desired instrument should draw a graph of torsional displacement against time. An ingenious electrical device has been developed by Dr. H. Thoma, Karlsruhe, Germany. The novel feature of this instrument was the use of toothed condenser plates, one attached to the crank shaft and the other to a uniformly rotating member. The variations of capacity of this condenser were used, as in the condenser microphone, to modulate a vacuum-tube circuit, the plate current of which passed through an oscillograph. This arrangement is capable of being calibrated and gives a displacement-time record. Another device which has been used successfully is Doctor Geiger's well-known torsiograph. It was by means of these instruments that the failures of the power plants on the *Graf Zeppelin* were analyzed.

An early design of torsiometer used by the Matériel Division incorporated links and pins to impart motion to an indicating hand. The latter moved over a scale graduated in degrees and was read by means of a neon stroboscope. Rapid wear proved to be fatal to this type of instrument, in spite of casehardened parts. In the spring of 1928 the idea occurred to the writer that a flexible cable of small size could be used, and this was accordingly incorporated in the instrument used during the summer of 1928. Figures 1 to 3 show general views of the instrument. The arrangement makes use of pulleys and a drum on the shaft carrying the pencil. This has proven very successful and is quite free of backlash. The cable eventually wears out but is readily replaced and, under the severest conditions so far met with, the life of the cable is several hours of continuous operation. The record is made on ordinary indicator paper. A drum is provided which retains the paper and since the pencil revolves with the instrument, the drum is held in the hand, inserted over the pilot provided for it and quickly pressed in and withdrawn. The last one-thirty-second inch of motion of the drum operates a bell-crank system and presses the pencil against the paper. A pointer is provided on the drum and with the engine set on top center of one cylinder, the beginning of the card is lined up with the pencil, the pointer being vertical. This serves to locate the crank positions on the records. The two revolutions in the engine cycle are superposed, hence the complete cycle crosses the card twice before it repeats. In the present design the records show a displacement of one-eighth inch for each 1° of torsional displacement of the crank shaft. The natural frequency of the spring-driven flywheel is 10 to 12 vibrations per second. This frequency is necessitated by the high accelerations to which aircraft engines are subjected. Figure 4 (d) shows the calibration record obtained at a speed of 30 r. p. m. The flywheel was plucked and released to obtain a record of the damped vibration of the spring-driven flywheel.

TEST RESULTS ON VARIOUS ENGINES

A number of records from various engines are presented in Figures 5 to 20. In Figure 5 is presented a series of cards taken with a standard Liberty metal propeller with blades set at an angle of 25° at 42″ radius. This held the full throttle speed down to 1,330 r. p. m. Two synchronous periods are evident, one at 1,000 r. p. m., card (c), and one at 1,330 r. p. m., card (j). In both cases the frequency is very close to 100 vibrations per second. At 1,000 r. p. m. we find 12 vibrations corresponding with the 12 power impulses per cycle or 6 per revolution. At 1,330 r. p. m. we find 9 vibrations per cycle, or 4½ per revolution. These two periods would be accounted for by the presence of twelfth and ninth harmonics in the single-cylinder torque curve. Cards (l) and (m) show the effect of cutting out one set of plugs. In the Liberty both plugs were near the center of the combustion space, hence little effect is observed other than the loss of 30 r. p. m. Card (n) shows the effect of retarded spark at full throttle. The speed dropped to 1,000 r. p. m., but since maximum pressure was reached later in the cycle, the deflections were greater than those in card (c). Vertical lines were drawn indicating approximately maximum torque (20° after top center) on the cylinders marked at the bottom. It is observed that all 12 vibrations at 1,000 r. p. m. and 6 out of 9 at 1,330 r. p. m., show maximum acceleration at about the time of maximum torque.

Figure 6 presents the records taken on the same engine with a propeller setting of 17° at 42″ radius. Here the maximum speed was 1,700 r. p. m., the rated speed of the Liberty "12." In this set of records three resonant periods are observed. That at 1,000 r. p. m. is of small amplitude, due to the small power impulses. The period 1,330 is apparent in cards (g) and (h), followed by a comparatively smooth range, cards (i) to (l). The third resonant period occurs at about 1,700 r. p. m., as shown in cards (o) and (p). The frequency is the same as at the two lower periods. It is to be noted that six out of seven of these vibrations show maximum acceleration close to points of maximum torque. This period would indicate a seventh harmonic in the resultant torque curves. No records have been taken at speeds in excess of the third resonant period on a Liberty 12-cylinder engine.

Figure 7 shows a set of cards taken on a Curtiss D–12 engine of about 1,150 cubic inch displacement. These were taken on the dynamometer, using a rather heavy moid coupling. The compression ratio of this engine was 7.3 to 1 and the engine was operated at the maximum load it was believed safe to carry at each speed up to 1,800 r. p. m. From this point cards were taken at full throttle up to 2,700 r. p. m. This was done in order to record the worst conditions obtainable in normal operation. Vertical lines indicate cylinders at top center. It is observed that definite resonance appears at 1,600 r. p. m., 2,100 r. p. m., and 2,700 r. p. m. These periods bear the same relation to each other as did 1,000 r. p. m., 1,330 r. p. m., and 1,700 r. p. m. in the case of the Liberty "12." If the frequency is accurately 160 vibrations per second, these resonance points should occur at 1,600 r. p. m., 2,130 r. p. m., and 2,740 r. p. m. It is evident from the records that 2,500 r. p. m. should be an excellent full throttle speed, since this would allow a throttle range of 400 r. p. m. before encountering resonance, reducing the power output on propeller load to 59 per cent of that at 2,500 r. p. m. It is doubtful if the effect of the resonant vibrations at 2,100 r. p. m. could be felt at this reduced output. In climbing, the engine might be called upon to operate within this critical range at full throttle, but the duration of such full throttle operation would not be great enough to cause crank-shaft failure. Unfortunately, no records are available on this engine with propeller load. It will be pointed out in connection with another set of cards that it is unsafe to draw conclusions from dynamometer records, due to the effect of the coupling used. In order to approximate propeller loading, the moment of inertia of the coupling should be large.

Figure 8 presents a very interesting record of a Curtiss 6-cylinder line engine of about 410 cubic inch displacement. In this engine, resonance with the power impulses could not be observed, since it would not occur until speeds in the neighborhood of 3,200 r. p. m. were reached. This is far beyond the operating range of the engine. There should also appear a range of resonance at a speed of about 2,130 r. p. m., but the available propellers would allow the engine to turn up only to 1,875 r. p. m. Cards (a) to (l) were taken on propeller load in the airplane, while cards (m) to (x) were taken on the dynamometer. In the latter case, a thermoid coupling was used to couple the engine to the dynamometer. Since it was of small size, its effect as a flywheel was negligible and the actual crank-shaft vibration was only slightly apparent in the records. These cards would seem to indicate that a coupling with appreciable hysteresis, such as that offered by the thermoid discs, might be a desirable type to incorporate in a geared engine. The chief objection to the thermoid coupling would be, of course, its size.

Figures 9 and 10 show cards taken on a Wright R–1750 9-cylinder radial engine. Figure 9 is from the direct drive engine and Figure 10 is from the 2:1 geared engine. The gearing is of the Farman type and does not incorporate any type of spring coupling. There is, however, indication of a considerable amount of flexibility in the reduction gearing, since the resonance period is lowered from about 2,100 r. p. m. to about 1,450 r. p. m. Assuming the same moment of inertia in both engines, the ratio of torsional stiffness from center of cylinders to edge of propeller in the direct engine is to that of the geared engine as 2100^2 to 1450^2, or 2.1 to 1. It is hard to say where this flexibility is introduced, as this type of reduction gear is very much like an automobile differential in which the spider drives the propeller and one of the bevel gears is held stationary, the other being driven by the engine. The frequency of the direct crank shaft is about 158 vibrations per second, while that of the geared shaft is reduced to about 109 vibrations per second. It is observed that torsional vibration occurs in the direct engine at normal operating speed of 1,900 r. p. m., while in the geared engine the operating range from 1,600 r. p. m. to 1,900 r. p. m. is free of resonant vibration. The range from 1,300 r. p. m. to 1,550 r. p. m. is practically never used. Hence this condition is satisfactory in the geared engine. In the case of the direct engine, however, stiffening of the crank shaft is indicated in order to eliminate possible trouble due to critical crank-shaft vibration.

Figure 11 shows records from the Pratt and Whitney Wasp and Hornet 9-cylinder radial engines, both of them direct drive. Both are free of critical vibration up to about 2,100 r. p. m., which is in excess of normal rated speed. The critical frequency of the first appears to be about 175 vibrations per second and of the other about 165 vibrations per second. These records constitute proof that the large radial engine can be so designed as to be free from trouble due to torsional vibration up to a speed as great as can be utilized before valve gear trouble makes itself felt and where inertia loading of the crank bearing becomes excessive.

Figure 12 is an interesting set of records from a Packard 2,500 series 12-cylinder **V** engine. Cards (a) to (k) were taken, using domestic aviation gasoline and 5½ c. c. of ethyl fluid. It is to be noted that while the period of the crank shaft is apparent at 1,350 r. p. m., the variations from cycle to cycle are much greater in amplitude. This was thought to be due to faulty distribution, hence the fuel was changed to highly volatile casinghead gasoline and cards (l) to (v) were taken. The engine operated much more smoothly on this gasoline, showing that a large part of the roughness of the engine was due to poor carburetor and manifold arrangement. It is to be observed that the crank shaft has a distinct torsional period at

1,350 r. p. m. and another at 1,800 r. p. m. Both of these show a frequency of 135 vibrations per second, there being 12 vibrations per engine cycle at 1,350 r. p. m. and 9 at 1,800 r. p. m. Since the rated speed of this engine is 1,800 r. p. m., the engine is called upon to operate for extended periods within this second critical range.

Figure 13 shows a set of records from the propeller end of the engine, using the same fuel as was used in cards (a) to (k), Figure 12. These records show quite definitely that deflections occurring at the rear end of the crank shaft do not indicate accelerations of the shaft and propeller as a unit. In resonant vibration, the shaft probably deflects according to an elastic curve, showing one node at a point close to the edge of the propeller hub. In this case, the torque on the most stressed section is nearly double that which, applied at the rear, would produce the same angular deflection as observed in operation. The large asynchronous deflections due to distribution may or may not follow this same type of elastic curve, depending on which cylinders receive the surges of pressure, or ramming effect of the manifold. The frequency of the latter effect in this engine, as indicated by vibrometer records, is from 12 to 21 per second. It is possible, therefore, that the actual deflection of cards (a) to (k), Figure 12, were even greater than shown by the records, since the instrument frequency was about 10 vibrations per second.

In connection with this test a second engine was available, which incorporated a reduction gear and an Allison type of spring coupling. It was also possible to replace some parts of this reduction gear with others and run without any spring coupling. Figure 14, cards (a) to (m), show the records obtained with the spring coupling, while (n) to (z) show those obtained with the solid gear drive. These latter records should bear some relation to those shown in Figure 12, (a) to (k). The fuel was the same and, aside from the gearing, the engines were alike. Little can be said as to frequency of vibration with the spring coupling, except to again observe that all speeds from 700 r. p. m. to 1,900 r. p. m. were about equally bad in respect to vibration, with no apparent tendency to damp out oscillations, as was observed in the case of the thermoid coupling, Figure 8, cards (m) to (x). It appears that the natural frequency of the coupling is about 55 vibrations per second, although it could have a variety of frequencies in view of its nonlinear deflection characteristic. Cards (n) to (z), without the spring coupling, show that much less vibration is found at all speeds. There appears to be some resonance at about 1,800 r. p. m. at a frequency of about 90 vibrations per second and also at 1,600 r. p. m. with frequency of about 99 vibrations per second. Cards (c), (e), (f), (y), and (z) show a form of vibration which has been found in two V engines with poor distribution. Here there are six vibrations per engine cycle, or three per revolution. There is no indication in the analysis by A. Stieglitz of the normal torque curve of a 12-cylinder V engine to indicate resonance with a sixth harmonic, as would be indicated by these records. If, however, due to faulty or uneven distribution, alternate power impulses were abnormal, a vibration of this type would occur, such as is observed in a 6-cylinder line engine, at resonance with the sixth harmonic. This form of vibration was observed on the air-cooled Liberty which has a rotary induction system and plain gallery manifolds. There is no reason why the air-cooled Liberty should differ greatly from the water-cooled in character of vibration, with the exception that pressure surges in the gallery type manifold are undoubtedly present, due to overlap of suction strokes. This form of vibration is not found on engines using four carburetor barrels, so that each barrel supplies three cylinders, at 240° intervals and forked rod construction.

Figure 15 is the record of the Curtiss V–1570 engine. Here resonance occurs at about 1,550 r. p. m. and 2,000 r. p. m. On the basis of a frequency of 155 vibrations per second, resonance would occur at 1,550 r. p. m., 2,060 r. p. m., and 2,660 r. p. m. This engine passed a 50-hour full throttle test at 2,400 r. p. m., but was found to have a cracked crank shaft at the conclusion of the test. Figure 16 is a set of records on the engine at conclusion of the test. The character of vibration is different from that observed with a perfect shaft, as might be expected. An attempt to run 50 hours at 2,600 r. p. m. terminated at 12½ hours with a broken crankshaft.

Figure 17 shows a set of records on the same engine as in Figures 15 and 16, but with 7:5 reduction gears of the spur type. A Curtiss spring coupling is incorporated, in which coil springs are under an initial compression and no motion of the coupling occurs until a definite torque is exceeded. The rate of increase of torque with deflection is also different after the coupling starts to yield than before. The torsional rigidity of the propeller crank-shaft system is very greatly reduced, giving a natural frequency of only about 80 vibrations per second. This frequency indicates the first torsional period at 800 r. p. m., the second at 1,070 r. p. m., the third at 1,370 r. p. m., and the fourth at 1,600 r. p. m. The third period at 1,370 r. p. m. does not appear in the records, but the fourth shows plainly at 1,600 r. p. m. This period, with three vibrations per revolution, is not caused by distribution, since this engine is particularly good in this respect. It would indicate a sixth harmonic for a 12-cylinder V engine, contrary to the analysis previously referred to by A. Stieglitz and reproduced in Figure 23. This engine, however, has the articulated rod construction, which may cause the torque curve from one bank to differ from the other.

In Figures 18 and 20 are presented some records taken on a Liberty "12" engine to study the effect of fuels, detonation and preignition on crank-shaft torsional vibration. Figure 18 shows the effect of changing the spark advance from 30° to 60°. This was, undoubtedly, accompanied by severe detonation, since the fuel was domestic aviation gasoline. Cards (o) and (s) serve to show the effect of premature ignition. Cards (t) to (w) show the effect of cutting out cylinder No. 1 or No. 6 combined with 30° and 60° advance. It appears that No. 6 (nearest the propeller) is the more serious loss, giving greater deflections than when cylinders near the rear end of the shaft cut out. It was then decided to try the worst fuel obtainable—low-test automobile gasoline, such as is used for washing purposes. To make matters as severe as possible, the plugs used were the hottest obtainable, and cards were then taken at 30° and 60° spark advance. The detonation must have been terrific, as evidenced by dense black puffs from the stacks. It is interesting to note, however, that no one was able to hear detonation above the other noises present. At another time silencers were fitted to a Liberty in the effort to make detonation audible, but without success. These silencers were effective enough to make unnecessary the use of ear defenders, and the fuel was low-test gasoline. However, no one could be sure he could detect audible detonation. When a Liberty single is running, exhausting into an exhaust duct, detonation may be heard for half a mile.

In Figure 19 are shown several cards taken under various conditions. These serve to show that premature ignition does increase the amplitude of torsional vibration, while detonation at normal spark advance does not affect torsional vibration. In case, however, of extended periods or detonation, hot spots may appear and cause preignition, and thus indirectly cause increased vibration. Figure 20 shows the best conditions obtainable. California gasoline was used with 11 cc. of ethyl fluid and cool plugs were installed. There were no visible signs of detonation, even with 60° spark advance. The surprising feature of this run is that the amplitude of torsional vibration is greater for a given spark setting and power output than when using low-test gasoline with preigniting spark plugs and extreme detonation.

Figure 23A and Figure 23B show comparative resonance deflections for 12-cylinder V and 6-cylinder line engines. Figure 23A is calculated on a frequency n=6,000, as found in the Liberty "12". These periods check with great accuracy the observed periods as indicated in Figures 5 and 6.

Figure 23B shows the 6-cylinder resonance curve on the basis of n=9,600 as found in the engine which gave the records shown in Figure 8. Unfortunately, only one of the periods predicted in Figure 23B could be observed.

These curves are reproduced from a paper on "Torsional Vibrations in Vertical Engines," by Albert Stieglitz, and published in Luftfahrtforschung, July 24, 1929.

METHOD OF CALCULATING FREQUENCY OF TORSIONAL VIBRATION

Several methods of attacking the problem of torsional vibration have been proposed and two of these in particular have given excellent results. B. C. Carter, in Engineering, July 13, 1928, has given an empirical formula based on numerous experiments. In his method, the crank is reduced to a torsionally equivalent length of shaft of the same cross section as the main journal. Prof. S. Timoshenko, in his book "Vibration Problems in Engineering," has given a rational method of reducing all cranks to a length of shaft having one torsional rigidity. Once having the crank replaced by a uniform shaft, and the inertia masses reduced to an equivalent flywheel system, the frequency of vibration is readily calculated. In aircraft work, the moment of inertia of the propeller is from ten to twenty times the total equivalent moment of inertia of the engine. It is, therefore, permissible to consider the nodal point as being located at the edge of the propeller hub. It could hardly be taken at the center of the hub, since the hub reinforces the shaft end and prevents it from deflecting. In the case of the radial engine, the equivalent moment of inertia of a given number of cylinders is approximately twice that of the line engines, because of the necessary balance weights. However, it is found that little error is introduced in considering the propeller moment of inertia as infinite; that is, assuming a node at the edge of the propeller hub.

Professor Timoshenko has stated in his book, "Vibration Problems in Engineering," that little error is introduced in calculating the primary or fundamental frequency, if all the equivalent flywheel masses at the crank pins are considered as adding together at the longitudinal center of symmetry of the engine. The problem is, therefore, reduced to that of finding the torsionally equivalent length of shaft from the edge of the propeller hub to the center of symmetry of the engine, and the equivalent moment of inertia of the crank-pin systems.

A typical aircraft engine crank shaft is shown in Figure 21. The notation used is given in Table 1 substantially as used by Timoshenko and Carter.

Table 1

a = actual length of crank pin	in.
a_1 = effective length of crank pin	$= a + 0.9h$ in.
$2b$ = actual length of main journal	in.
$2b_1$ = effective length of main journal	$= 2b + 0.9h$ in.
B_2 = flexural rigidity of crank pin	$= I_2E = 1.474 \times 10^6 (D^4_2 - d^4_2)$
B_3 = flexural rigidity of crank cheek	$= I_3E = 2.5 \times 10^6 hw^3$
C_1 = torsional rigidity of main journal	$= J_1G = 1.178 \times 10^6 (D^4_1 - d^4_1)$
C_2 = torsional rigidity of crank pin	$= J_2G = 1.178 \times 10^6 (D^4_2 - d^4_2)$
C_3 = torsional rigidity of crank cheek	$= J_3G = 3.33 \times 10^6 w^3h^3/(w^2 + h^2)$
D_1 = outside diameter of main journal	in.
d_1 = inside diameter of main journal	in.
D_2 = outside diameter of crank pin	in.
d_2 = inside diameter of crank pin	in.
E = Young's Modulus of elasticity for steel	$= 30 \times 10^6$ lb./in.^{2}
F_2 = cross sectional area of crank pin	$= \frac{\pi}{4} (D^2_2 - d^2_2)$
F_3 = cross sectional area of crank cheek	= hw
G = Modulus of elasticity in shear or torsion	$= 12 \times 10^6$ lb./in.^{2}
h = thickness of crank cheek	in.
I_1 = moment of inertia of main journal section	$= \frac{\pi}{64} (D^4_1 - d^4_1)$ in.^{4}
I_2 = moment of inertia of crank pin section	$= \frac{\pi}{64} (D^4_2 - d^4_2)$ in.^{4}
I_3 = moment of inertia of crank cheek section	$= hw^3/12$ in.^{4}
J_1 = polar moment of inertia of main journal section	$= 2I_1$ in.^{4}
J_2 = polar moment of inertia of crank pin section	$= 2I_2$ in.^{4}
J_3 = polar moment of inertia of crank cheek section	$= w^3h^3/3.6(w^2 + h^2)$ in.^{4}
K = factor of complete constraint by main journals (Timoshenko) = $\dfrac{\frac{R(a+h)^2}{4C_3} + \frac{aR^2}{2C_2} + \frac{a^3}{24B_2} + \frac{R^3}{3B_3} + \frac{1.2}{12 \times 10^6}\left(\frac{a}{2F_2} + \frac{R}{F_3}\right)}{\frac{aR}{2C_2} + \frac{R^2}{2B_3}}$	in.
L = length of connecting rod	in.
l_1 = equivalent length of shaft from edge of propeller to first crank throw minus b	in.

l_2 = equivalent length of shaft, center to center of main journals if shaft is unconstrained .... $= 2b_1 + a_1 \frac{C_1}{C_2} + 2R \frac{C_1}{B_3}$ in.

l_3 = equivalent length of shaft, center to center of main journals if shaft is constrained by main bearings. $= 2b_1 + a_1 \frac{C_1}{C_2}\left(l - \frac{R}{K}\right) + 2R \frac{C_1}{B_3}\left(l - \frac{R}{2K}\right)$ .... in.

$L_f = l_1 + \frac{n}{2} \times l_2$ (unconstrained) .... in.

$L_c = l_1 + \frac{n}{2} \times l_3$ (constrained) .... in.

m_1 = rotating weight per crank pin .... lb.

m_2 = reciprocating weight per crank pin .... lb.

n = number of crank throws.

R = crank radius .... in.

w = width of crank cheek .... in.

The equivalent moment of inertia of one crank system is, according to Professor Timoshenko:

$$i = \left[m_1 + \frac{m_2}{2}\left(l + \frac{R^2}{4L^2}\right)\right]R^2 + i_c$$

Since the average ratio of crank to connecting rod is about 1/4, this value of $\frac{R}{L}$ may be used, making the term $\frac{1}{2}\left(l + \frac{R^2}{4L^2}\right)$ equal to 0.508. This may be used for all practical purposes, hence we may take for the whole engine—

$$I = n[(m_1 + 0.51 m_2)R^2 + i_c] \text{ ..... lb.in.}^2 \quad (1)$$

in which i_c = calculated or experimentally determined value of the moment of inertia of one crank, about the main journal center, lb. in.²

The frequency of oscillation of a torsion pendulum is $f = \frac{1}{2\pi}\sqrt{\frac{C}{I}}$ in which C is the torque in lb. ft. per radian, required to deflect the torsion member, and I is the moment of inertia of the pendulum in slug ft.² In design calculations, it is convenient to use C_1 in lb. in. per radian per inch length and I in lb. in.², in which case $C = \frac{C_1}{L}$ and $f = \frac{1}{2\pi}\sqrt{\frac{12 \times 32.2 \times C_1}{IL}}$ or $3.125\sqrt{\frac{C}{IL}}$

For the unconstrained case, $f_f = 3.125\sqrt{\frac{C_1}{IL_f}}$ (2)

For complete constraint, $f_c = 3.125\sqrt{\frac{C_1}{IL_c}}$ (3)

Major Carter's method, see Engineering (British), July 13, 1928, is purely an empirical one, and is given only for the case of the shaft in its bearings. The equivalent length of shaft, from center to center of main journals, is given as—

$$l_3 = (2b + 0.8h) + \frac{3}{4}\left(\frac{D^4_1 - d^4_1}{D^4_2 - d^4_2}\right)a + \frac{3R}{2}\left(\frac{D^4_1 - d^4_1}{hw^3}\right) \quad (4)$$

in which the symbols have the same meaning as before.

The above methods will now be applied to the Liberty "12" engine, where—

a = ... 2.5 in.

$a_1 = 2.5 + 0.9 \times 1$ = ... 3.4 in.

$2b$ = ... 2 in.

$2b_1 = 2 + 0.9 \times 1$ = ... 2.9 in.

$B_2 = 1.474 \times 10^6 (2.375^4 - 1.25^4)$ = ... 43.1×10^6

$B_3 = 2.5 \times 10^6 \times 1 \times 3.4375^3$ = ... 101.2×10^6

$C_1 = 1.178 \times 10^6 (2.625^4 - 1.375^4)$ = ... 51.8×10^6

$C_2 = 1.178 \times 10^6 (2.375^4 - 1.25^4)$ = ... 34.5×10^6

$C_3 = 3.33 \times 10^6 \times 3.4375^3 \times 1^3/(3.4375^2 + 1^2)$ = ... 10.6×10^6

D_1 = ... 2.625 in.

d_1 = ... 1.375 in.

D_2 = ... 2.375 in.

d_2 = ... 1.25 in.

$F_2 = \frac{\pi}{4}(2.375^2 - 1.25^2)$ = ... 3.2 in.²

$F_3 = 1 \times 3.4375$ = ... 3.4375 in.²

h = ... 1 in.

w = ... 3.4375 in.

m_1 = ... 6.3 lb.

m_2 = ... 12.4 lb.

i_c = ... 92.5 lb. in.²

n = ... 6

R = ... 3.5 in.

$$K = \frac{\frac{3.5(2.5+1)^2}{4 \times 10.6 \times 10^6} + \frac{2.5 \times 3.5^2}{2 \times 34.5 \times 10^6} + \frac{2.5^3}{24 \times 43.1 \times 10^6} + \frac{3.5^3}{3 \times 101.2 \times 10^6} + \frac{1.2}{12 \times 10^6}\left(\frac{2.5}{2 \times 3.2} + \frac{3.5}{3.44}\right)}{\frac{2.5 \times 3.5}{2 \times 34.5 \times 10^6} + \frac{3.5^2}{2 \times 101.2 \times 10^6}} = \text{ ..... } 9.35 \text{ in.}$$

l_1 = ... 7.25 in. (actual) or 7.6 in. (equivalent length)

$l_2 = 2.9 + 3.4\frac{51.8 \times 10^6}{34.5 + 10^6} + 2 \times 3.5\frac{51.8 \times 10^6}{101.2 \times 10^6}$ = ... 11.58 in.

$l_3 = 2.9 + 3.4\frac{51.8 \times 10^6}{34.5 \times 10^6}\left(1 - \frac{3.5}{9.35}\right) + 2 \times 3.5\frac{51.8 \times 10^6}{101.2 \times 10^6}\left(1 - \frac{3.5}{2 \times 9.35}\right)$ = 9 in.

$L_f = 7.6 + 3 \times 11.58$ = ... 42.36 in.

$L_c = 7.6 + 3 \times 9$ = ... 34.6 in.

$I = 6[(6.3 + 0.51 \times 12.4)\,3.5^2 + 92.5]$ = ... 1,482 lb. in.²

$f_f = 3.125\sqrt{\frac{51.8 \times 10^6}{1482 \times 42.36}}$ = ... 90 vib./sec.

$f_c = 3.125\sqrt{\frac{51.8 \times 10^6}{1482 \times 34.6}}$ = ... 99.5 vib./sec.

According to Major Carter—

$l_3 = (2 + 0.8 \times 1) + \frac{3}{4}\left(\frac{2.625^4 - 1.375^4}{2.375^4 - 1.25^4} \times 2.5\right) + \frac{3 \times 3.5}{2}\left(\frac{2.625^4 - 1.375^4}{1 \times 3.4375^3}\right)$ = ... 11.53 in

$L = 7.6 + 3 \times 11.53$ = ... 42.19 in.

$f = 3.125\sqrt{\frac{51.8 \times 10^6}{1482 \times 42.19}}$ = ... 90 vib./sec.

The observed value is about 100 vib./sec. as shown by the records, Figures 5 and 6.

Table 2 shows the calculated and observed frequencies for several engines:

Table 2

Engine	Timoshenko		Carter	Observed	
	Unconstrained	Constrained		Frequency	Crit. r. p. m.
Liberty "12," 5" x 7" Vee	(1) 90	100	90	100	1,000, 1,330, 1,715
12-cyl. 4½" x 6" Vee (Curtiss D-12)	(2) 147.5	161	152	160	1,600, 2,130, 2,710
12-cyl. 5⅛" x 6¼" Vee (Curtiss V-1570)	(3) 139.3	153	149.5	155	1,550, 2,070, 2,660
6-cyl. 4½" x 5½" line (Curtiss Crusader)	(4) 151	161	155	160	1,600, 2,140, 3,200
OX5, 8-cyl. Vee	(5) 132	135.5	133.5	(1)	[1] 2,036
12-cyl. 6⅜" x 6½" Vee (Packard 2A-2500)	(6) 129	137	138	135	1,350, 1,800, 2,320
12-cyl. 6⅜" x 6½" Vee (Packard 3A-2500 geared)	(7) 70.5	71.8	71.8	(2)	700-1,900

[1] Not observed. [2] Ext./vib.

File D 00.12/122/No. 3533 (C.A.C.I.C. 1)

AIR CORPS INFORMATION CIRCULAR

Vol. VII, No. 664

Change)
No. 1)

War Department, Air Corps
April 20, 1936.

Page 6, Columns 1 and 2, of Air Corps Information Circular Vol. VII, No. 664, "Vibration Characteristics of Aircraft Engine Crank Shafts", is corrected by direction of the Chief of the Air Corps, in accordance with a recommendation from the Materiel Division contained in letter dated April 9, 1936, as follows:

Column 1

Line 4 reads $(\ell - \frac{R}{K})$ should read $(1 - \frac{R}{K})$

Line 5 reads $(\ell - \frac{R}{2K})$ should read $(1 - \frac{R}{2K})$

Line 15 reads $(\ell + \frac{R^2}{4L^2})$ should read $(1 + \frac{R^2}{4L^2})$

Line 18 reads $(\ell + \frac{R^2}{4L^2})$ should read $(1 + \frac{R^2}{4L^2})$

Column 2

Line 29 reads $\frac{51.8 \times 10^6}{34.5 + 10^6}$ should read $\frac{51.8 \times 10^6}{34.5 \times 10^6}$

(C.A.C.I.C. No. 1, April 20, 1936.)

U-1164,A.C.,4/21/36

METHOD OF CALCULATING FREQUENCY IN RADIAL ENGINES

In applying the foregoing calculations to radial engines, it is found that Professor Timoshenko's unconstrained case applied with sufficient accuracy. This is probably because in the usual radial engine designs the after-crank throw is supported by only one bearing and the shaft has a certain degree of freedom. The equivalent shaft length is calculated only to the center line of the engine, hence the after-crank cheek need not be considered. The crank shaft is counterbalanced in all single-row radial engines, hence its moment of inertia may readily be found by treating it as a compound pendulum, the only data necessary being the crank radius, reaction of counterbalances at crank radius, and time of one swing right to left about the main journal axis. The moment of inertia of the shaft is then:

$$i_e = W_e R t^2 \frac{144 g}{12 \pi^2} = 39.2\ W_e R t^2 \text{ lb. in.}^2 \qquad (5)$$

Where W_e = weight of counterweights at crank radius..lb.
R = crank radius........................in.
t = time of one-half vibration (one swing)..sec.

No appreciable error is introduced in thus including the shaft end which projects into the hub.

The equivalent moment of inertia of the rotating and reciprocating parts is, as in the line engine—

$$i_p = (m_1 + 0.51 m_2) R^2 \text{......lb. in.}^2) \qquad (6)$$

Figure 22 shows the crank shaft of a Pratt and Whitney Wasp 9-cylinder radial engine of 1,340 cubic inches displacement, in which—

$\frac{a}{2}$ = (center of engine to crank cheek) = ... 1.78 in.
$\frac{a_1}{2} = 1.78 + 0.45 \times 1.475 =$ .... 2.44 in.
b = ... 0.4375 in.
$b_1 = 0.438 + 0.45 \times 1.475 =$ .... 1.098 in.
$B_3 = 2.5 \times 10^6 \times 1.475 \times 3.75^3 =$ .... 194.3×10^6
$C_1 = 1.178 \times 10^6\ (2.874^4 - 1.656^4) =$ .... 71.7×10^6
$C_2 = 1.178 \times 10^6\ (2.685^4 - 1.125^4) =$ .... 59.1×10^6
D_1 = ... 2.874 in.
d_1 = ... 1.656 in.
D_2 = ... 2.685 in.
d_2 = ... 1.125 in.
h = ... 1.475 in.
w = ... 3.75 in.
m_1 = (rotating weight of master and link rods) = ... 19.8 lb.
m_2 = (reciprocating weight of pistons and rods) = ... 56.55 lb.
W_e = weight of counter weights at crank radius = ... 48.27 lb.
R = ... 2.875 in.
t = time of one swing of crankshaft = ... 0.517 sec.
$i_e = 39.2 \times 48.27 \times 2.875 \times 0.517^2 =$ .... 1,460 lb. in.²
$i_p = (19.8 + 0.51 \times 56.55)\ 2.875^2 =$ .... 402 lb. in.²
$I = i_e + i_p =$ .... 1,862 lb. in.²
l_1 = (actual, since torsional rigidity approximately uniform) .... 8.4 in.

$$L_f = l_1 + b_1 + \frac{a_1}{2}\frac{C_1}{C_2} + R\frac{C_1}{B_3} =$$

$$8.4 + 1.098 + 2.44\frac{71.7 \times 10^6}{59.1 \times 10^6} + 2.875\frac{71.7 \times 10^6}{194.3 \times 10^6} = \text{.......... 13.56 in.}$$

$$f_f = 3.125\sqrt{\frac{71.7 \times 10^6}{1862 \times 13.56}} = \text{.......... 166 vib./sec.}$$

The cricital r. p. m., therefore, will be for 9 cylinders,

$$N = \frac{166 \times 60}{4.5} = 2{,}210 \text{ r. p. m.}$$

Figure 11, (a) to (n), shows the torsiometer records on this engine at throttling loads from 2,400 r. p. m.; also, full throttle from 1,875 r. p. m. to 2,375 r. p. m.

Figure 11, (aa) to (nn), shows the records of a Pratt and Whitney Hornet 9-cylinder radial engine of 1,860-cubic-inch displacement on propeller load. The calculated frequency is 180 vibrations per second and the critical speed

$$N = \frac{180 \times 60}{4.5} = 2{,}400 \text{ r. p. m.}$$

A similar calculation on a Kinner 5-cylinder engine of 370-cubic-inch displacement shows the frequency to be 156 vibrations per second and critical speed

$$N = \frac{156 \times 60}{2.5} = 3{,}750 \text{ r. p. m.}$$

No test records are available on this engine.

Table 3

Engine	Timoshenko (unconstrained)	Observed	Critical r. p. m.
R-1750 (Wright Cyclone)	162.5	143-165	2160
GR-1750 (Wright Cyclone)	....	101-116	1450
R-1860 (Pratt and Whitney Hornet)	180	158-180	2400
R-1340 (Pratt and Whitney Wasp)	166	154-170	2210
R-370 (Kinner K-5)	156	....	3750

Figure 4 (a) and (b) show torsiometer records of the nose end of a Liberty "12," using two types of propeller, metal and wood. A small displacement unundoubtedly occurs, but due to the relatively large moment of inertia it is insignificant; hence in aircraft work, it appears sufficiently precise to record only the motion of the rear end of the crank shaft. These records also serve to show the desirability of driving accessories, and particularly superchargers, from the propeller end of the shaft, where torsional vibrations are of low amplitude. Slipping clutches, spring couplings, etc., could then be omitted, making a saving in weight. Ignition and valve timing would also be more uniform, resulting in smoother operation of the engine.

Card (c), Figure 4, shows a method of arriving at the accelerations which occur in actual accessory drives, using the conventional rear-end drive.

From the foregoing, it is seen that calculation of torsional periods from design data is sufficiently precise to make such a study worth while in any new design as well as in existing engines. Data are not available on all types of engines, but the leading types have been covered by experiment, so that the character of vibration to be expected is fairly well established.

RECOMMENDATIONS

It is recommended that the Matériel Division and Power Plant Branch continue this study until all types of engines and reduction gears have been analyzed.

BIBLIOGRAPHY

Vibration Problems in Engineering.... S. Timoshenko
D. Van Nostrand Co.

Empirical Formula for Crankshaft Stiffness in Torsion.............................. B. C. Carter
Engineering (British) July 13, 1928.

Critical Speeds of Crankshafts ---------- A. Gorfinkel
Engineering (British) Dec. 27, 1929.

Torsional Vibration in Vertical Engines --- A. Stieglitz
Luftfahrtforschung, July 24, 1929.

Inertia Torque ------------------ Author not named
Automobile Engineer, September, 1929.

Vibration Characteristics of Standard Liberty Engine ---------------------------------- F. L. Prescott
A. C. Technical Report, serial No. 3106.

Effect of Fuels, Detonation, and Preignition on Torsional Vibration ------------------- F. L. Prescott
A. C. Technical Report, serial No. 3147.

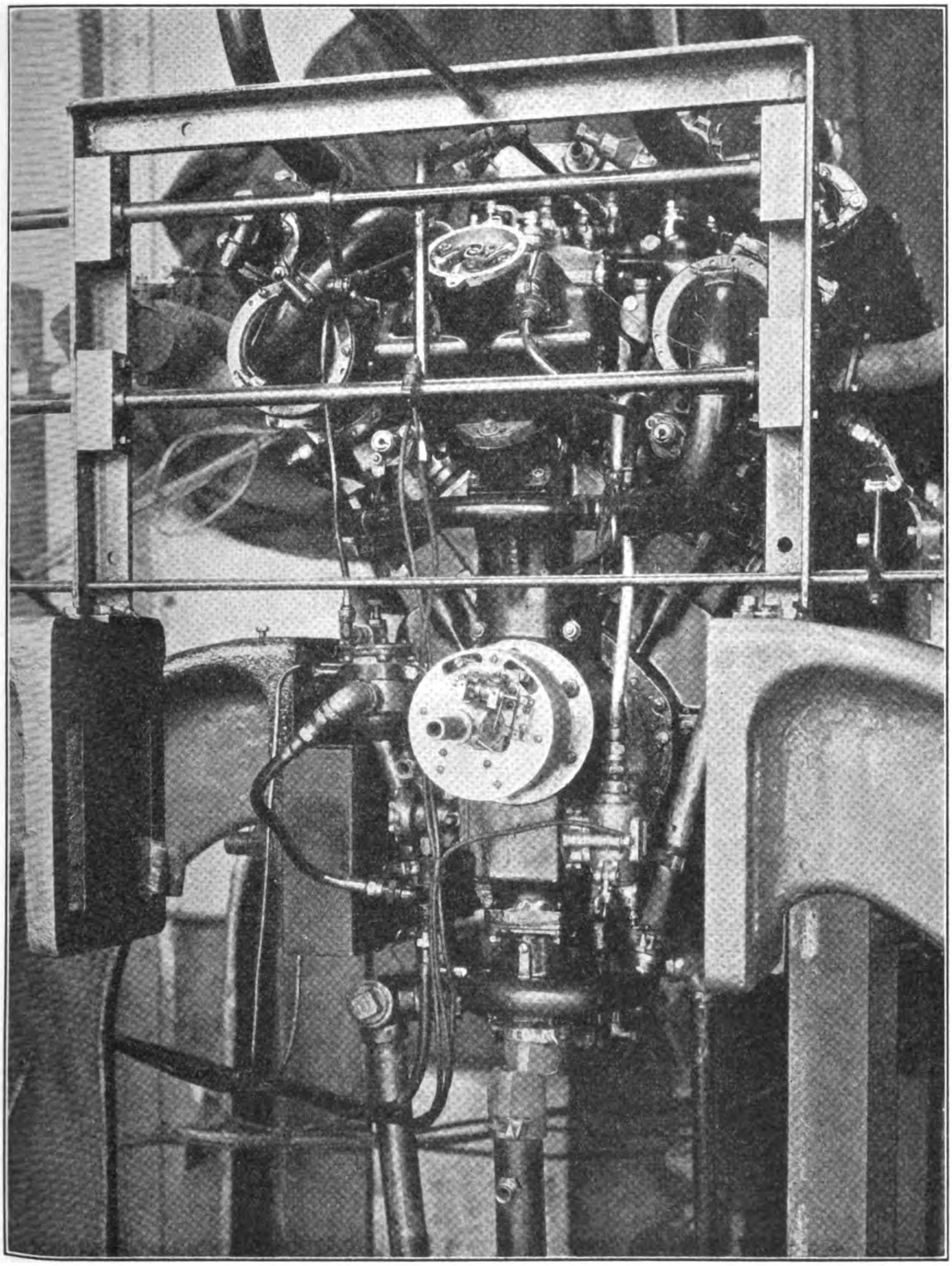

FIGURE 1.—General view of Matériel Division torsiometer mounted on a V-1570 engine in the laboratory

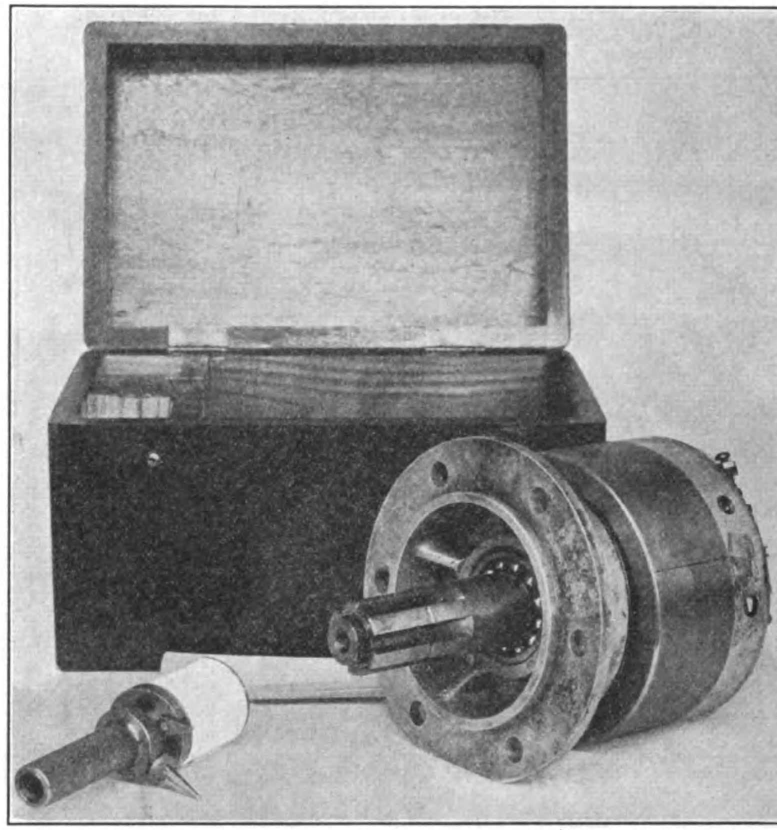

FIGURE 2.—View of Matériel Division torsiometer showing method of attaching to engine; and showing expanding spline for attachment to starter claw

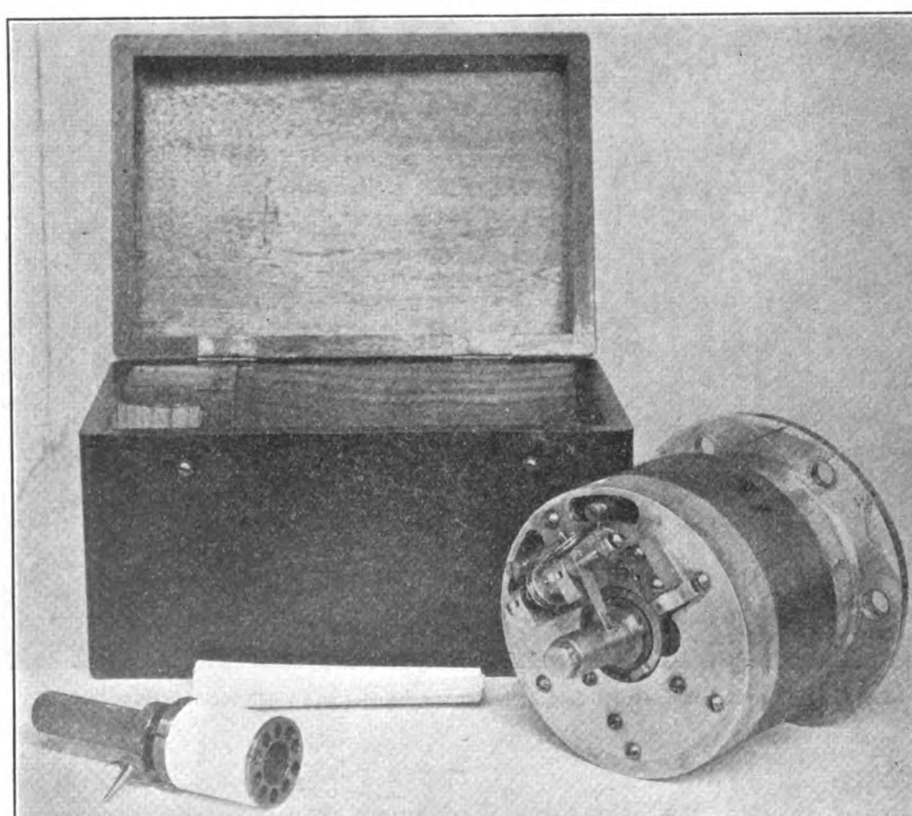

FIGURE 3.—View of Matériel Division torsiometer showing details of recording mechanism; also showing operating mechanism

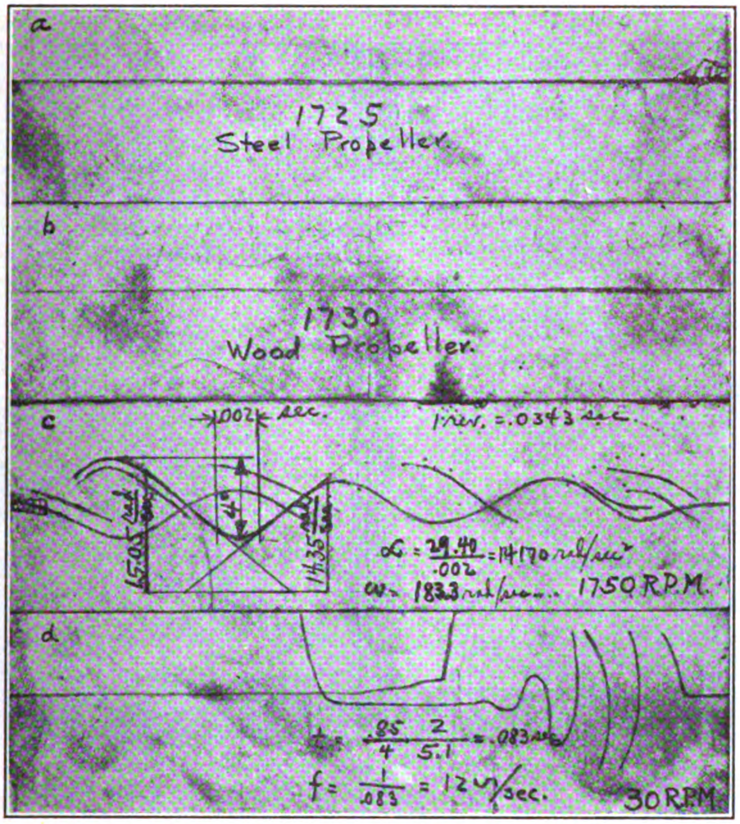

FIGURE 4.—(a) and (b) Torsiograms of propeller end of Liberty "12" with steel and wood propellers. (c) Method of calculating acceleration from records. (d) Method of determining natural period of instrument

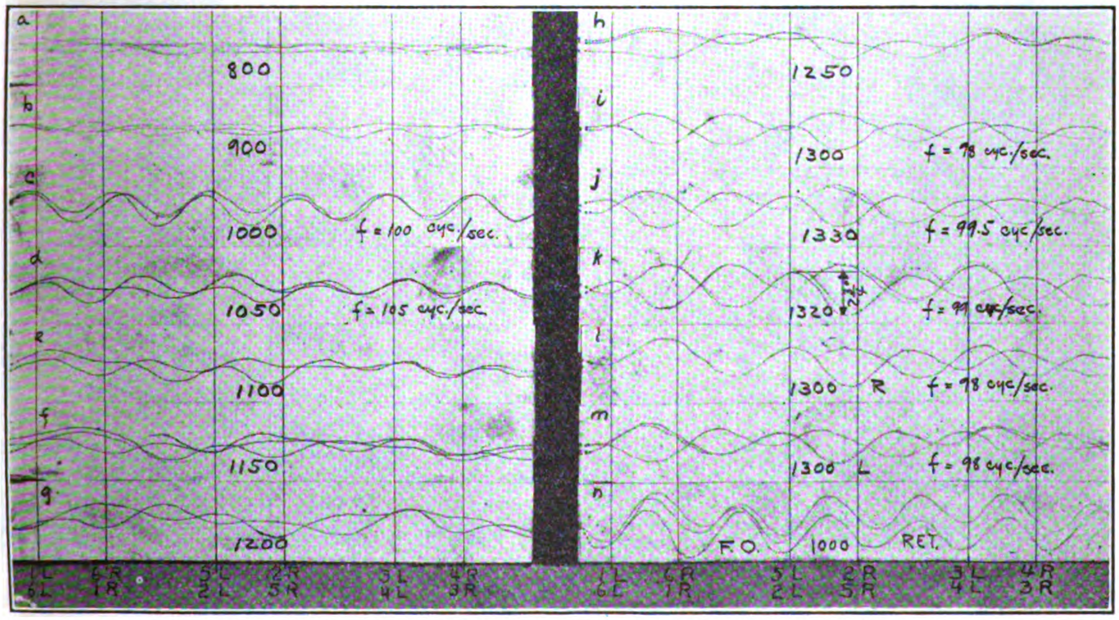

FIGURE 5.—Records from Liberty "12" with metal propeller set 25° at 42" radius

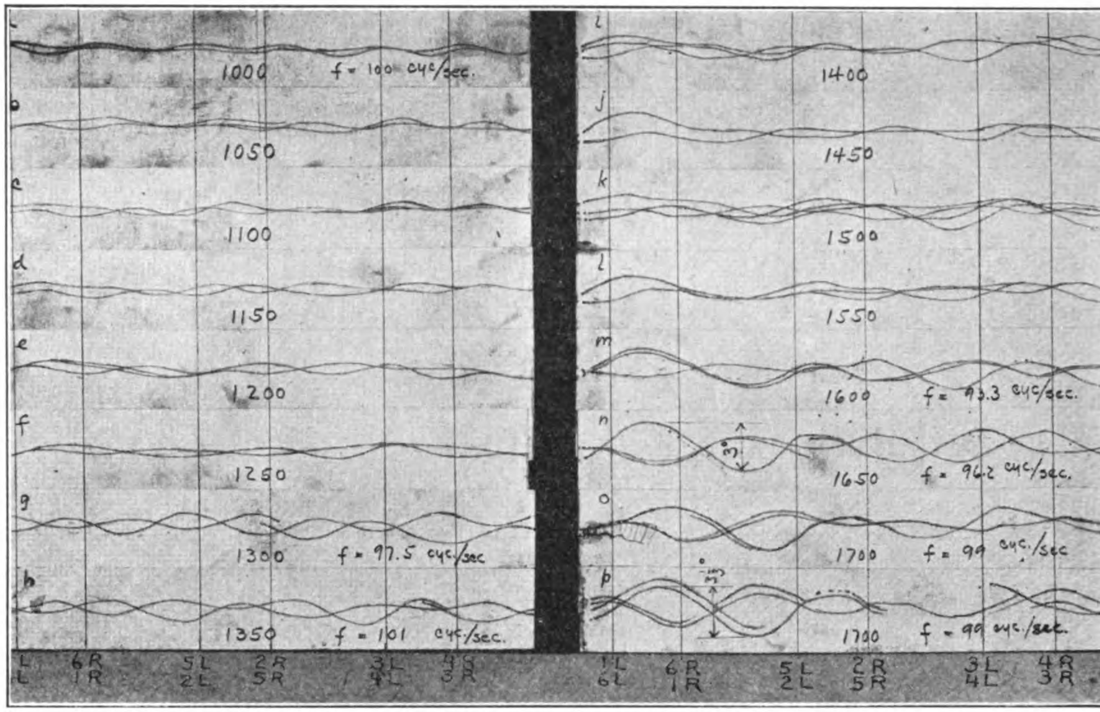

FIGURE 6.—Records from Liberty "12" with metal propeller set 17° at 42" radius

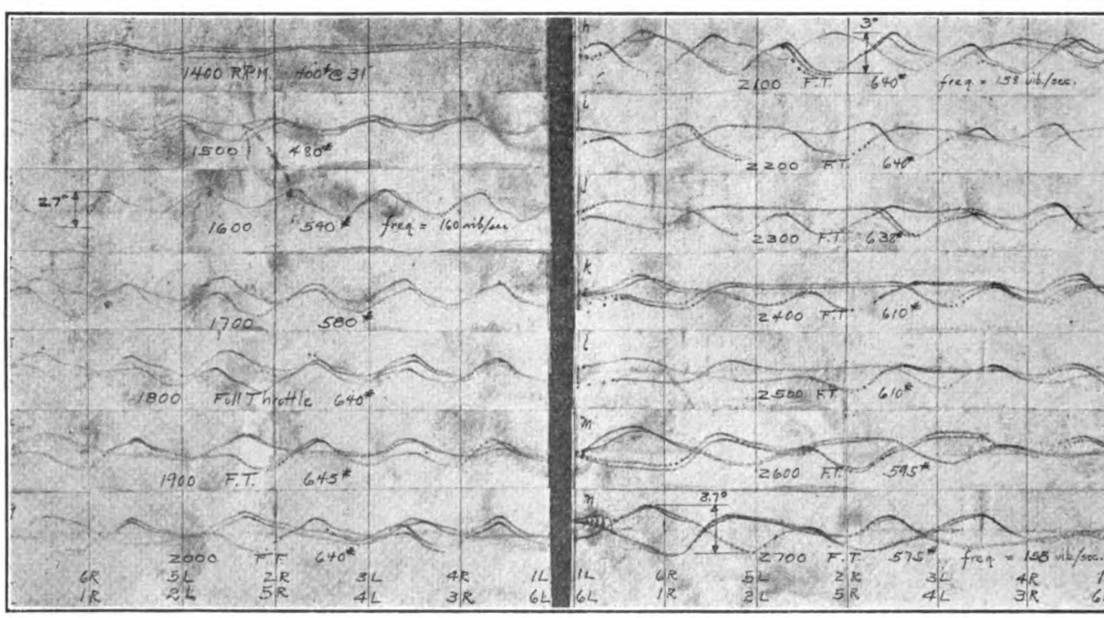

FIGURE 7.—Records from Curtiss D-12 on dynamometer, 7.3 compression ratio. Full throttle from 1,800 to 2,700 r. p. m.

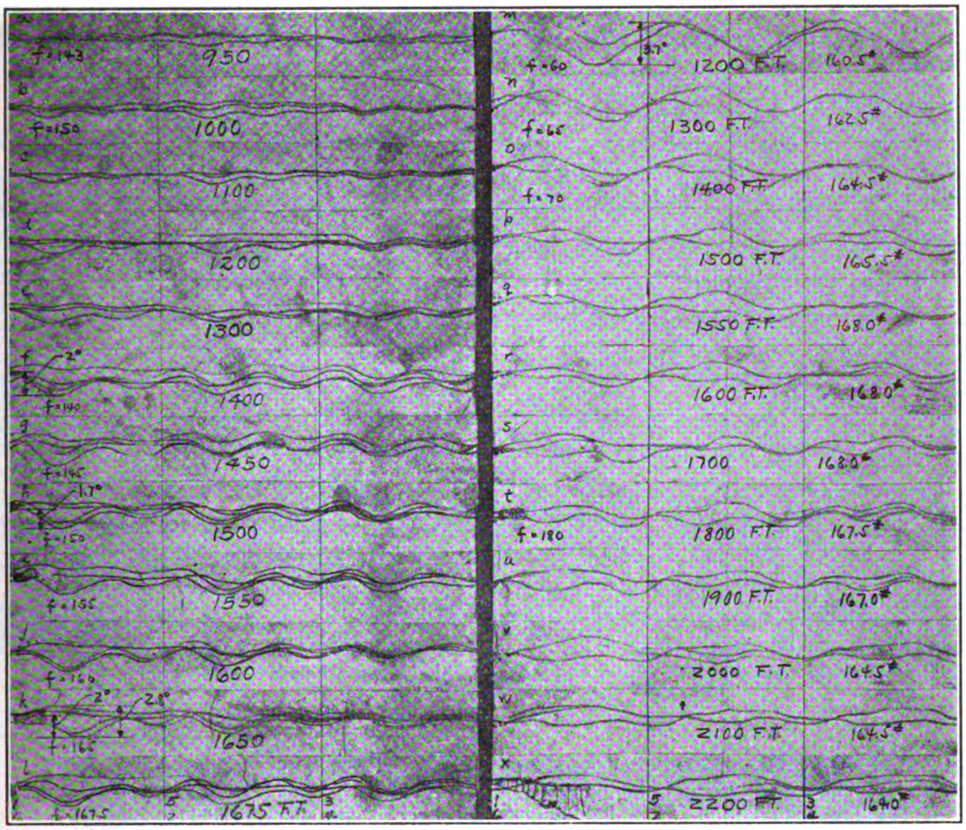

FIGURE 8.—Records from Curtiss Crusader 6-cylinder in-line engine (*a*) to (*l*) on propeller load. (*m*) to (*x*) on dynamometer using Thermoid coupling

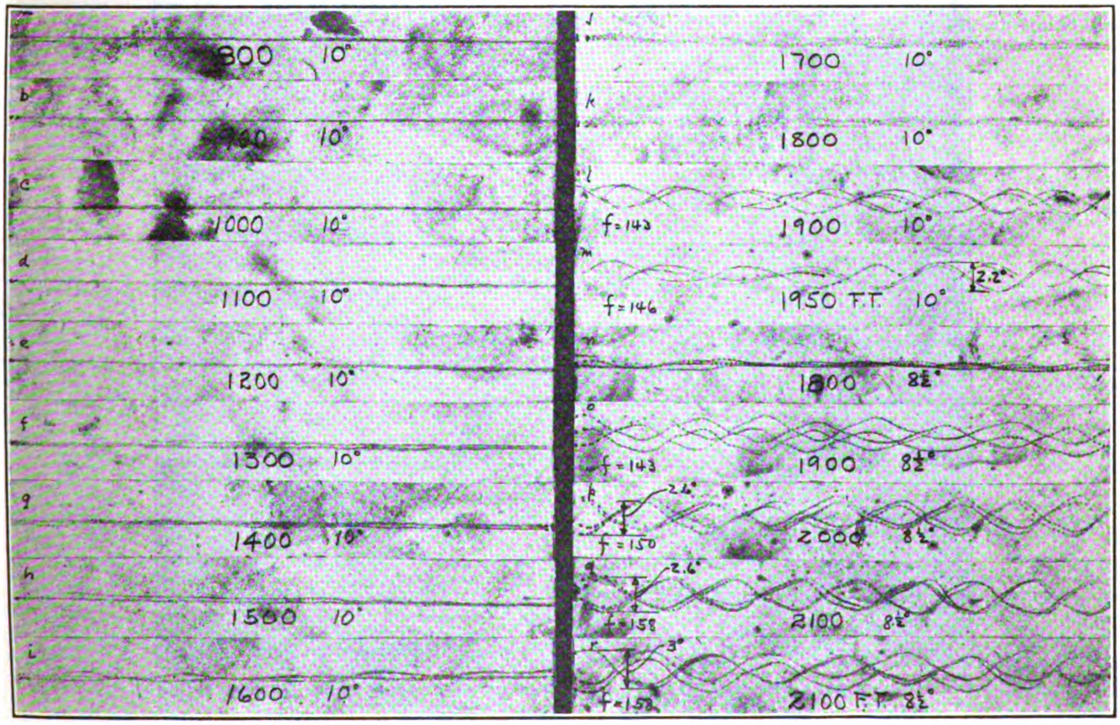

FIGURE 9.—Records from R-1750 (Wright Cyclone) standard propeller set 10° and 8½°

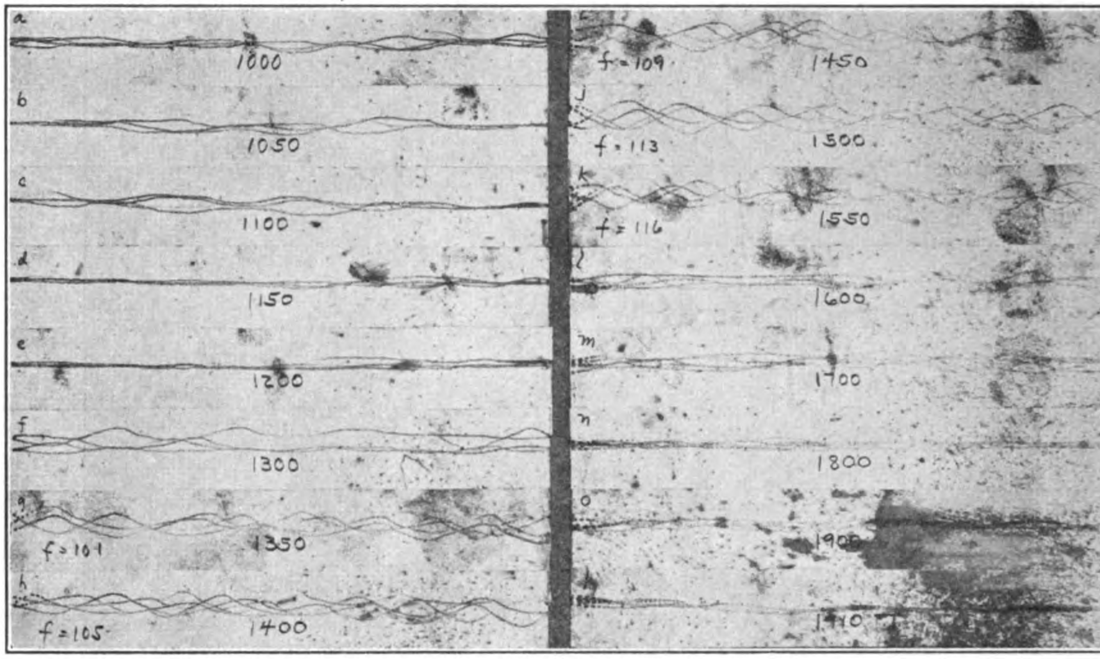

FIGURE 10.—Records from GR-1750 (2:1 Geared Wright Cyclone) propeller load

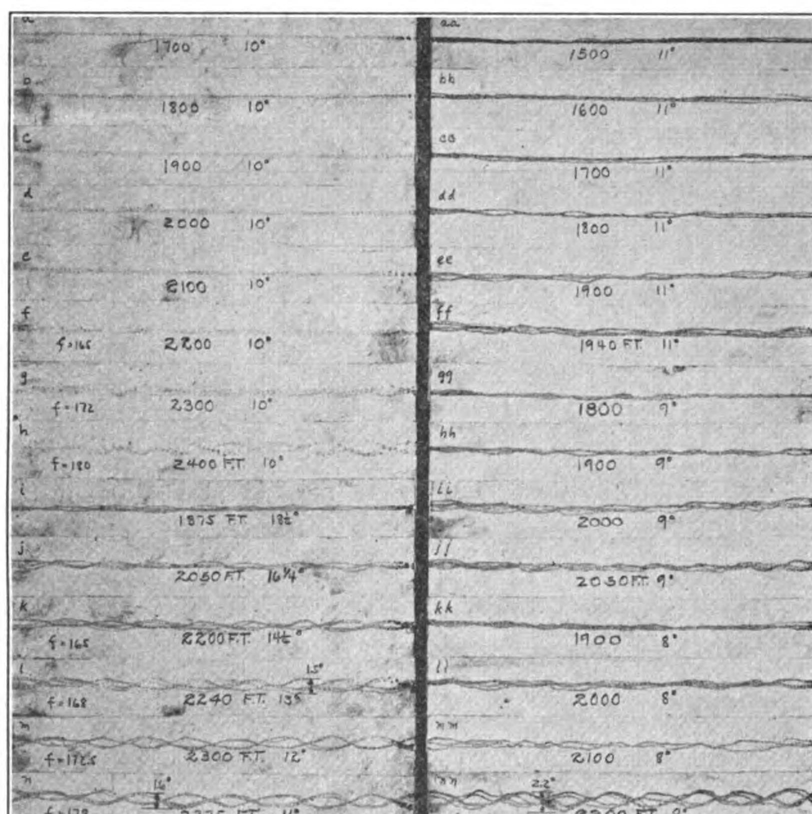

FIGURE 11.—(*a*) to (*n*) records of R-1340 (P. & W. Wasp) propeller settings 10° to 18¼°. (*aa*) to (*nn*) records of R-1860 (P. & W. Hornet) propeller settings 8° to 11°

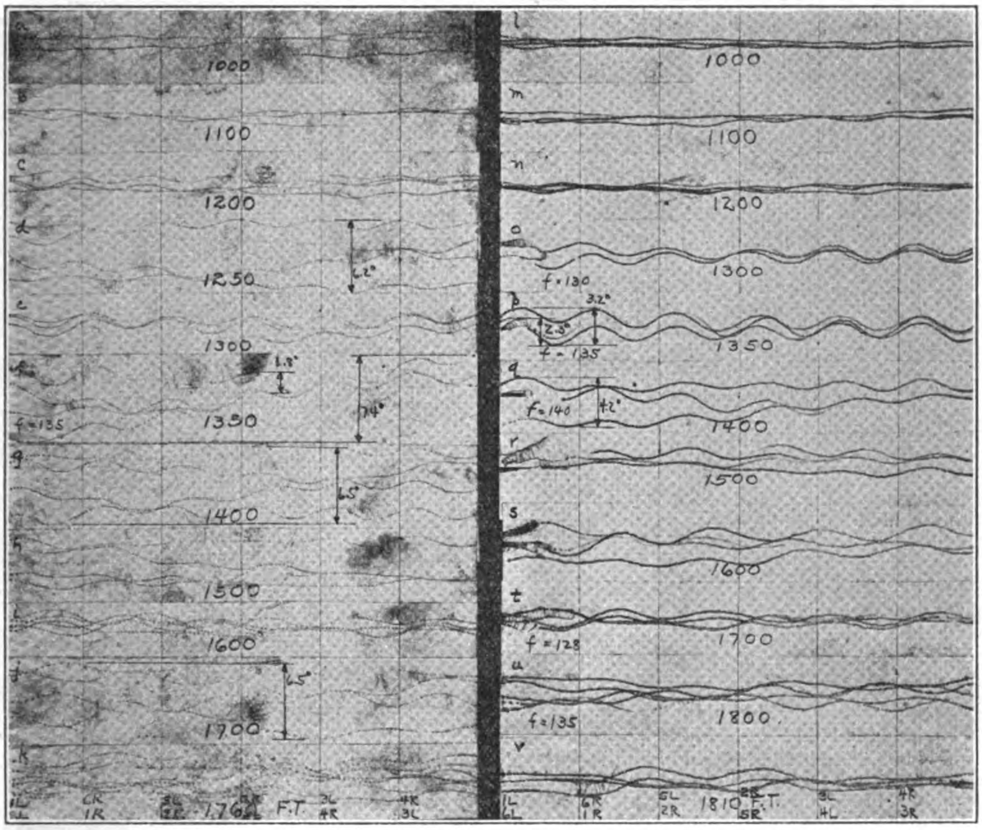

FIGURE 12.—Records of 2A-2500 (Packard) direct drive engine. (*a*) to (*k*) with domestic aviation gasoline and 5½ cc. ethyl fluid. (*l*) to (*v*) with casing-head gasoline

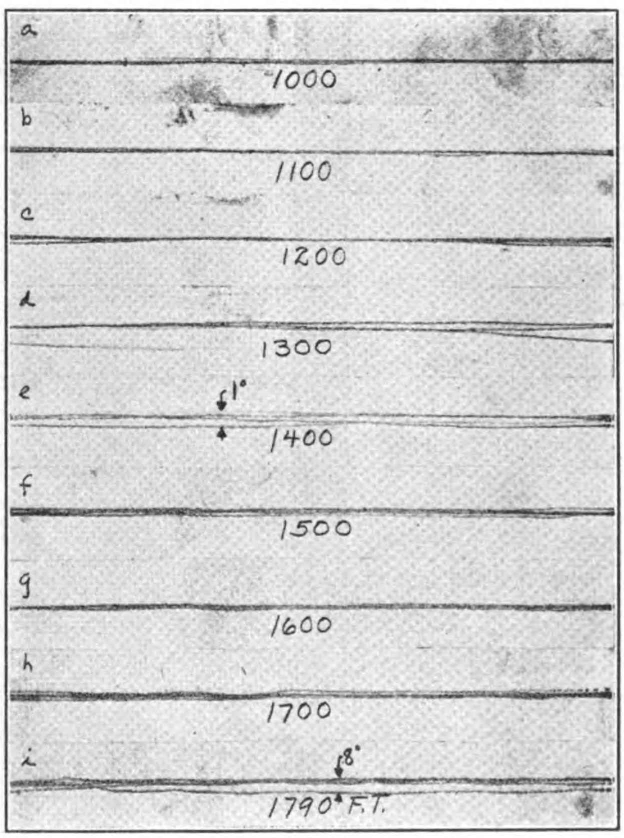

FIGURE 13.—Records of propeller end of shaft 2A-2500 (Packard) direct drive engine, same conditions as in Figure 11 (*a*) to (*k*)

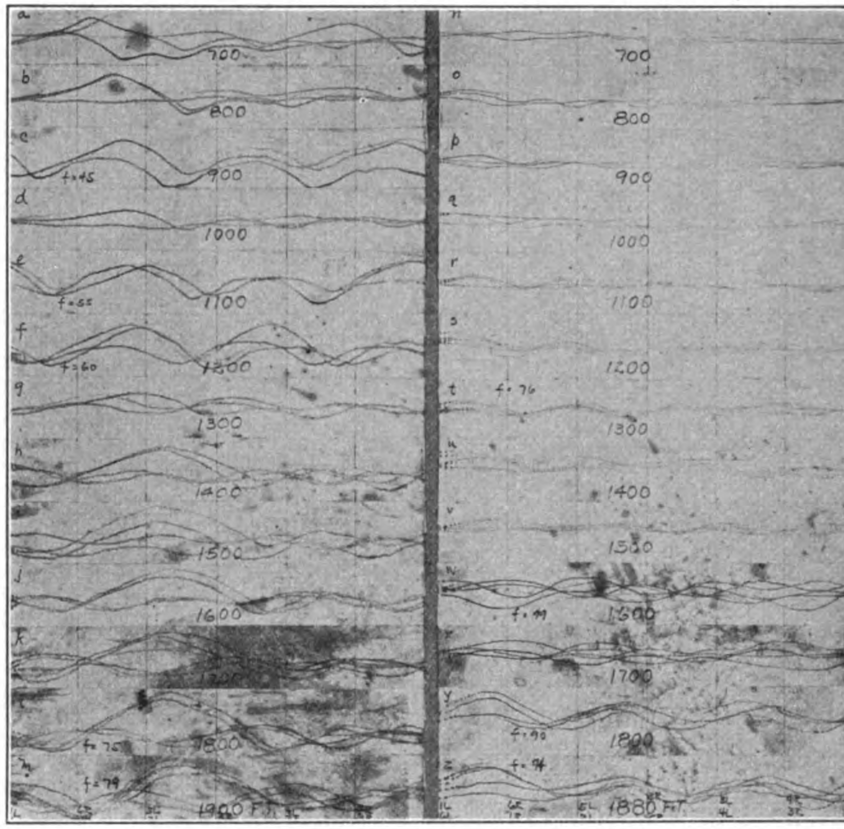

FIGURE 14.—Records of 3A-2500 (Packard) geared engine. (*a*) to (*m*) with Allison spring coupling. (*n*) to (*z*) with rigid gear drive

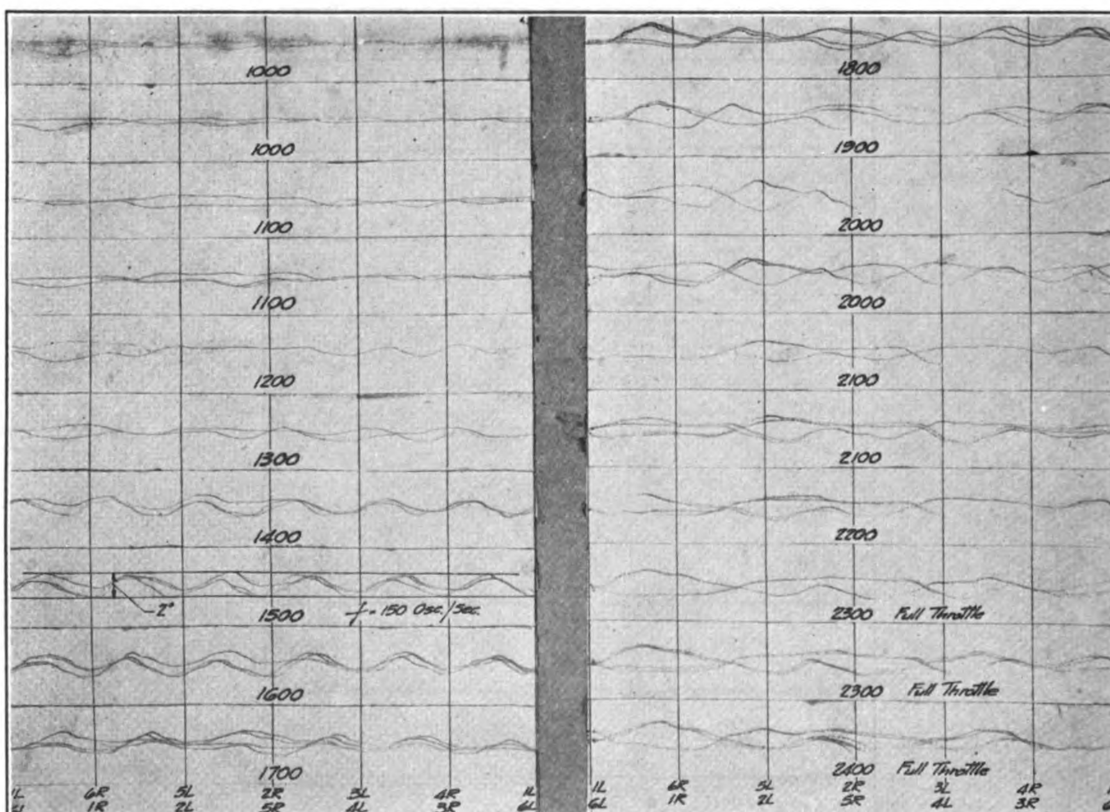

FIGURE 15.—Records of V-1570 (Curtiss Conqueror) on dynamometer using Thermoid coupling. "Propeller Load" from 2,300 r. p. m.

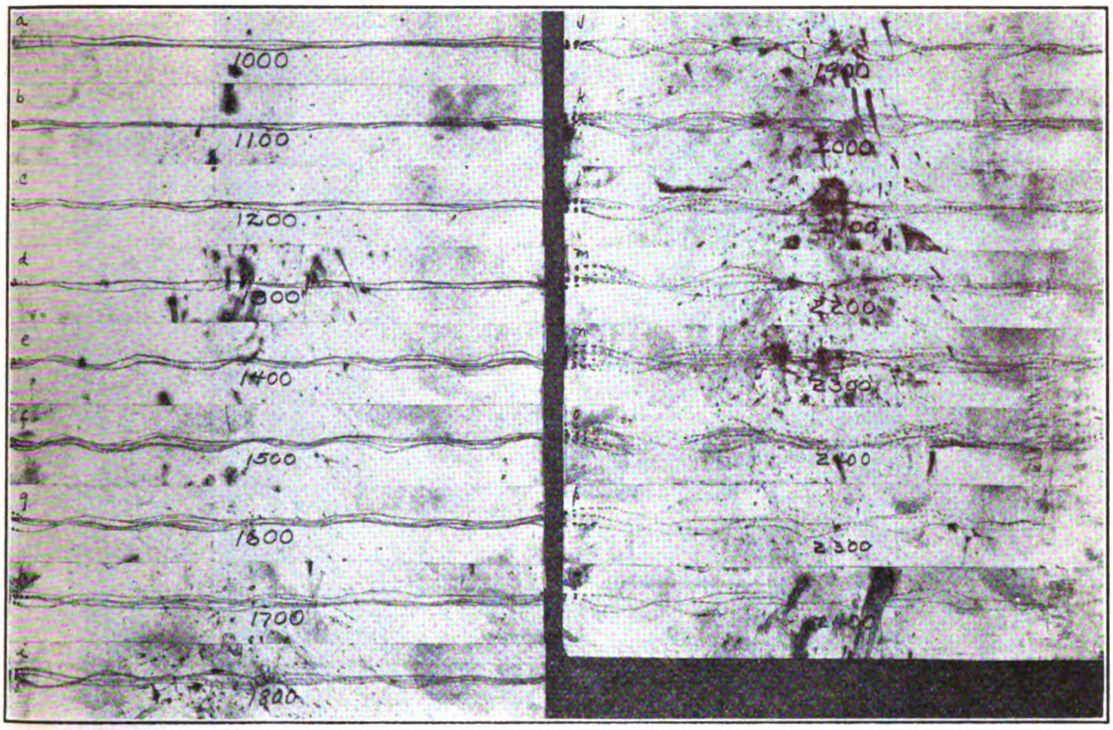

FIGURE 16.—Records of V-1570 (Curtiss Conqueror) at conclusion of 50 hours full throttle at 2,400 r. p. m. Crank shaft cracked at center main journal

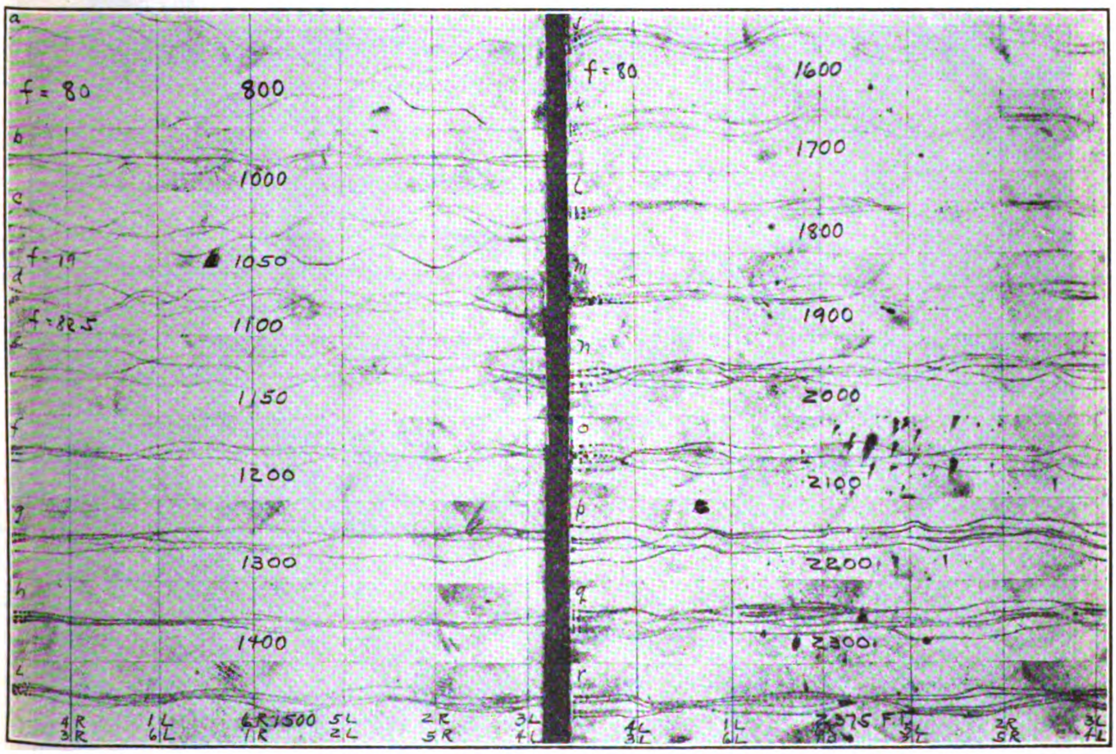

FIGURE 17.—Records of GV-1570 (Curtiss Conqueror) 7:5 gears, Curtiss spring coupling, propeller load

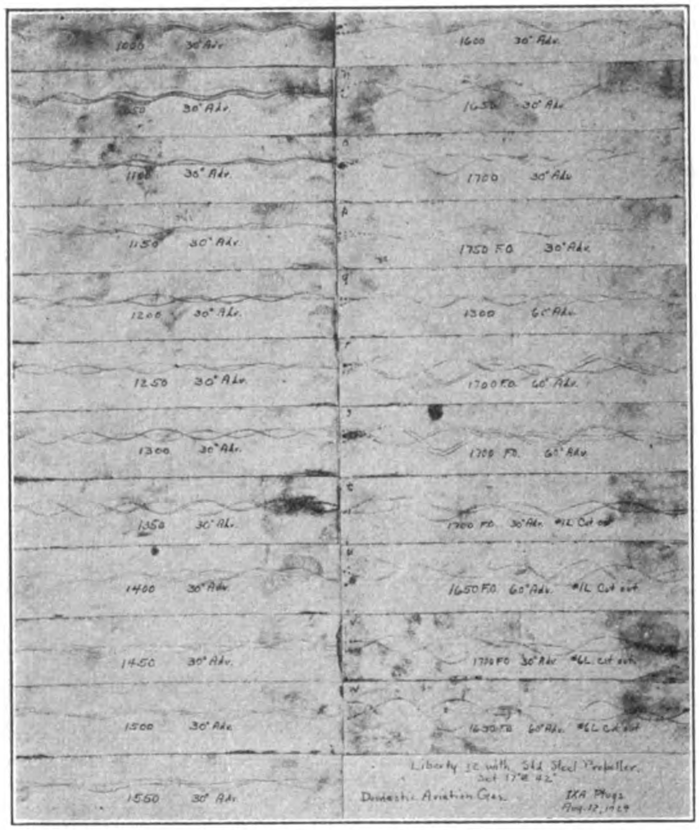

FIGURE 18.—Records of Liberty "12" showing effect of 30° and 60° spark advance, and effect of cutting out cylinders

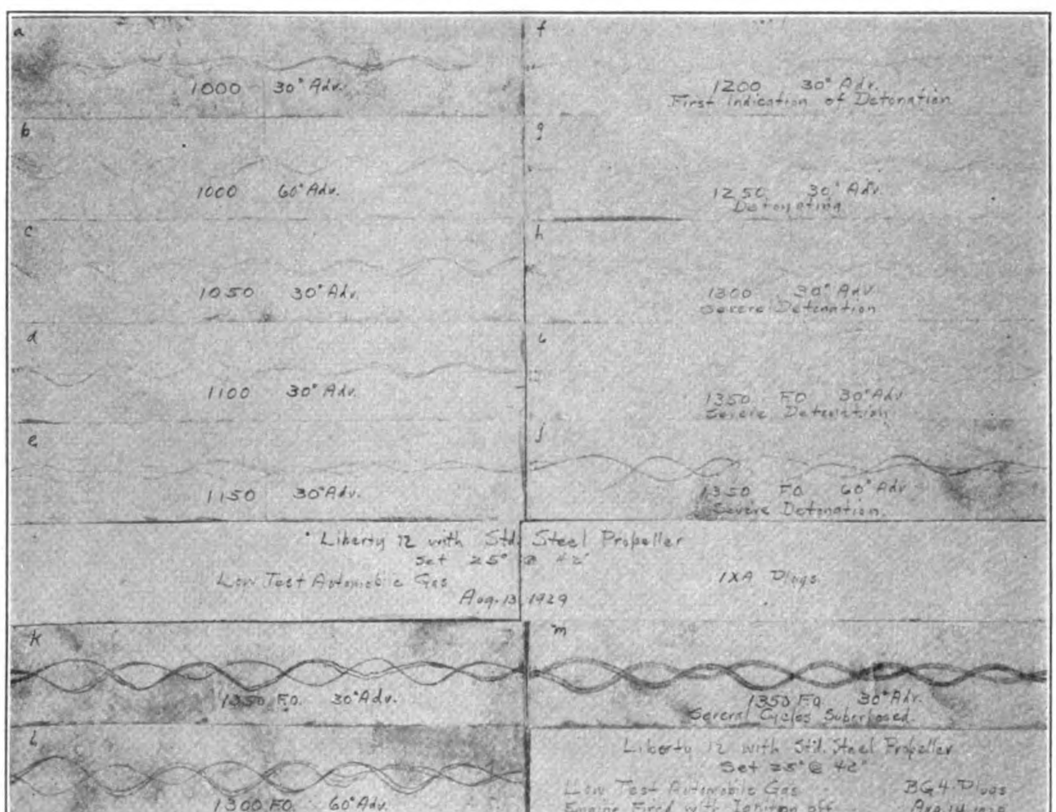

FIGURE 19.—Records of Liberty "12" with low-test gas, cool and hot plugs, and 30° and 60° spark advance (severe detonation)

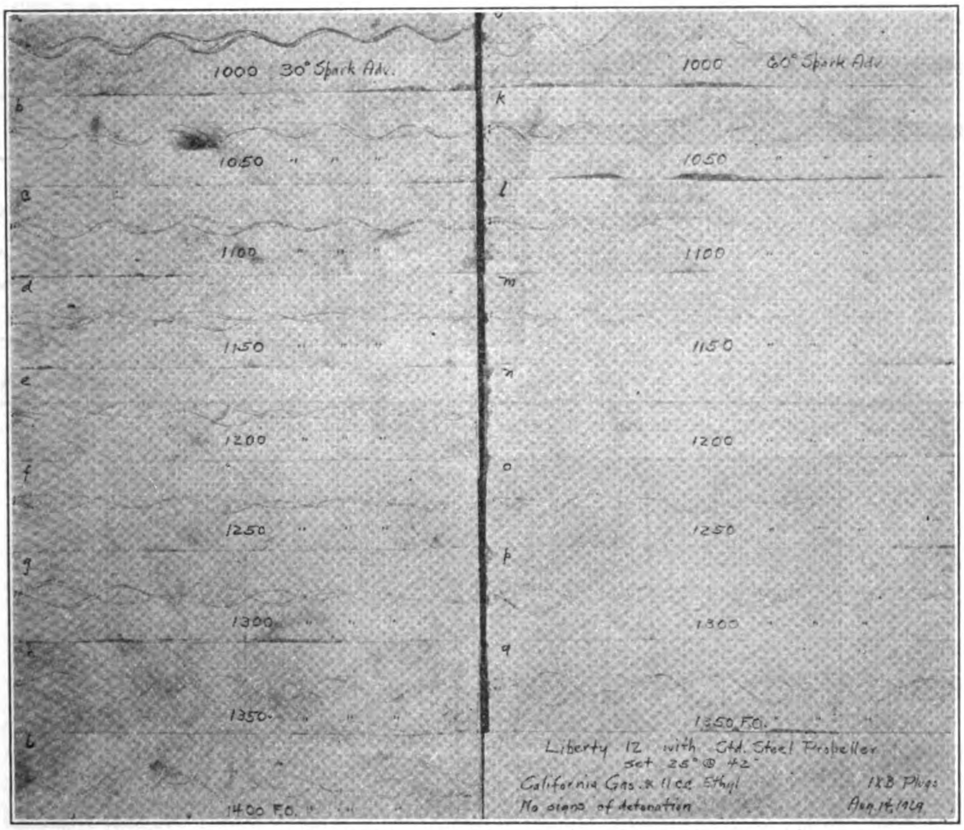

FIGURE 20.—Records of Liberty "12" with California gasoline and 11 cc. ethyl fluid, 30° and 60° spark advance, cool plugs. No detonation

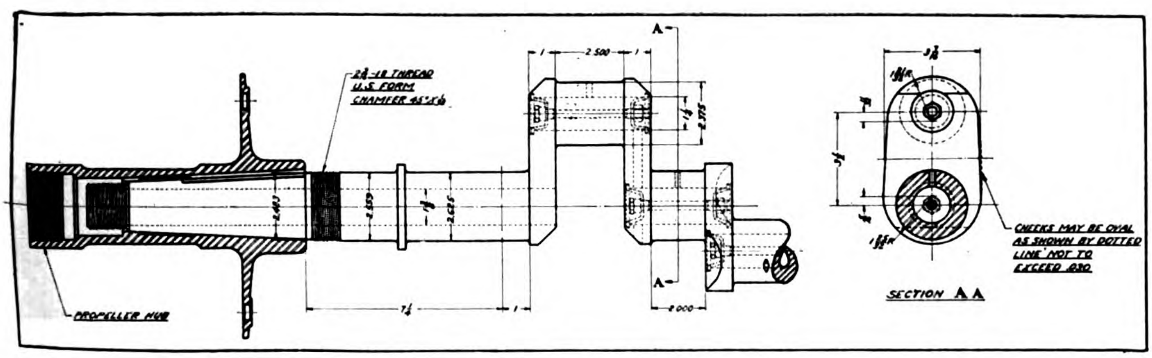

FIGURE 21.—Detail of Liberty "12" crank shaft

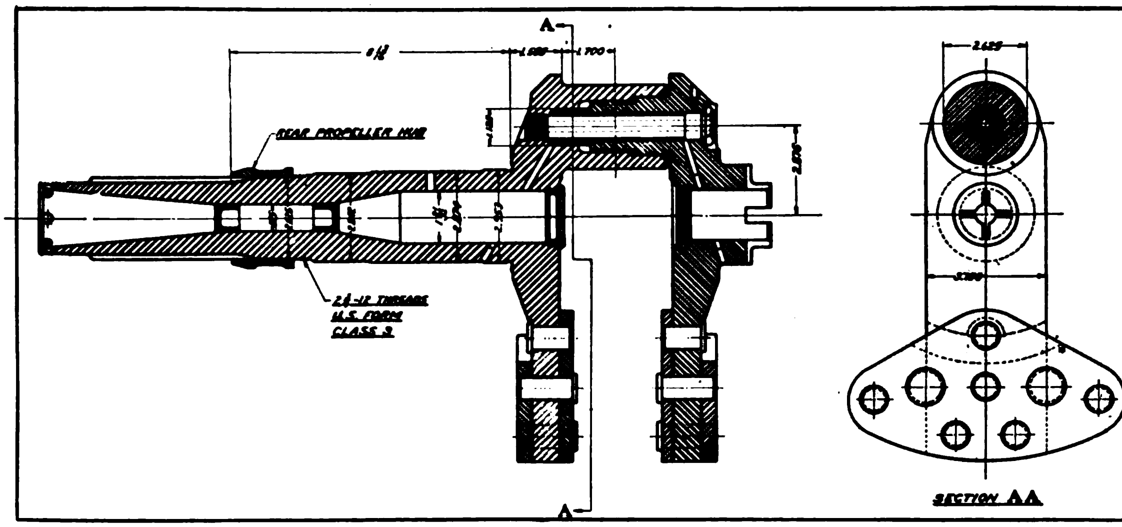

Figure 22.—Detail of Wasp (R-1340) crank shaft

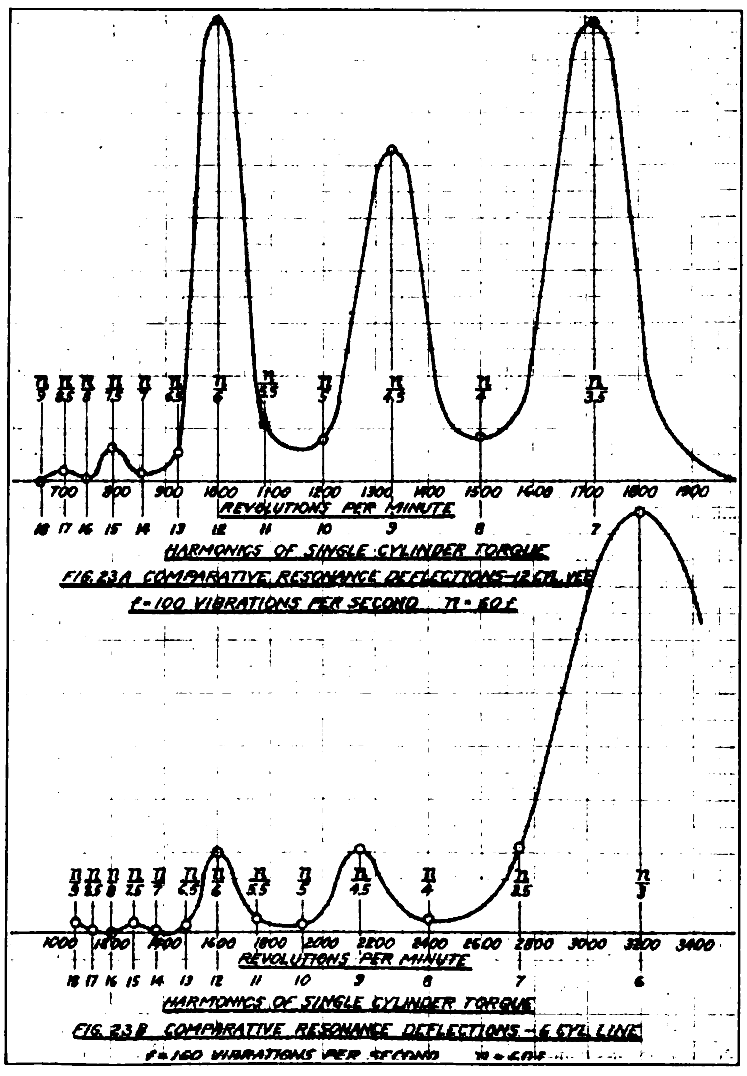

Figure 23 A and B—Comparative resonance deflections 12-cylinder Vee and 6-cylinder in-line engines