Torsional Vibration of Aircraft Engine Crankshafts

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Results of the study of torsional vibration of aircraft engine crankshafts are presented. Almost all engines have one or more easily detectable critical speeds for torsional crankshaft vibration. If one such critical speed should fall in the operating range of the engine, it is likely to cause trouble. Many apparently unrelated failures of engine parts can be traced to crankshaft vibration. It is therefore important that the vibration characteristics of every engine model be known so as to intelligently develop its design for maximum reliable power output.

THE subject of torsional vibration of crankshafts is certainly not new. Neither is the mathematical analysis of natural frequencies and critical speeds as much of a mystery to engineers as it was some years ago. Almost all phases of the subject have been discussed in the technical literature of recent years, and it therefore becomes difficult to present any new data which will be of interest to any one who has studied this source of trouble in internal-combustion engines. However, since aircraft engines appear to hold a popular interest and since they are by no means immune to the malady under consideration, a short presentation of some of the results of such vibration studies may be in order.

Almost all engines have one or more easily detectable critical speeds for torsional crankshaft vibration. If one such critical speed should fall in the operating range of the engine, it is very likely to cause trouble. There is a popular misapprehension to the effect that if the crankshaft of an engine does not break during operation, the engine must necessarily be free from serious crankshaft vibration. Such is not always the case. A long list of apparently unrelated engine-parts failures can be traced to crankshaft vibration, because such vibration besides overstressing the crankshaft also induces abnormal stresses in crankcases, connecting rods, valve gear, and accessory drives—particularly supercharger drives. It is therefore of great importance that the vibration characteristics of every engine model be known in order to intelligently develop its particular design for maximum reliable power output.

Present-day conventional aircraft engines can be classified roughly under two headings: first, in-line engines and, second, radial engines. Most of the engines under the first classification will be found to be liquid cooled, while the majority under the second classification are air cooled. However, as far as this study goes the method of cooling is of no great importance and will be given only little attention. The vibration characteristics of the engines under these classifications are as distinct as the classification to which they belong, and it is this point and the reason therefor that we wish to consider among other things.

Before taking up any specific phase of this subject, however, let us review briefly the underlying cause of torsional vibration in any engine in order to direct our attention intelligently and in order to give us a consistent method of attack in all cases.

Every crankshaft and connecting-rod system has at least one natural mode of vibration, the frequency of which can be determined. The calculation of natural vibration frequencies has been treated so thoroughly in technical literature of recent years that it need only be mentioned here. Any one not already familiar with the subject need only refer to Professor Timoshenko's books or to F. M. Lewis's paper of 1925, in order to get the necessary information for calculation of natural vibration frequencies. Only the single-noded mode of vibration will be considered in this discussion, since that is almost invariably the one which gives trouble. Since we have at our disposal, then, ready means for determining natural frequencies of crankshaft assemblies, we need only in addition to determine the engine speeds at which dangerous resonance may occur. Again we find ample literature to help us out, and we will only consider the main points.

As we all know, torsional crankshaft vibration is caused by torque variation. Every engine has a torque curve—sometimes unfortunately so. This torque curve is made up of the summation of the turning efforts of all cylinders combined in the proper phase relation as determined by the firing order. The fluctuations indicated by this curve may be large or small, depending upon the engine; but regardless of the size of the engine, the curve will repeat itself once every two revolutions of the crankshaft in the case of a conventional four-stroke-cycle engine. Accepting the harmonic method of analysis, we can take the turning-effort curve for each cylinder unit and resolve it into its harmonic components, the magnitude of which we are able to determine. We can then combine each successive component of each cylinder with the corresponding components of all the other cylinders of the engine in their proper phase relation and determine which harmonic of the single-cylinder torque curve will have a resultant effect in the engine. Selecting from this analysis the harmonic components which are cumulative, we can easily establish the location of our critical speeds. If we are still curious and wish to try to calculate the amplitude of vibration at a critical speed, it becomes necessary to know the magnitude of the damping forces in the engine, and in our present state of ignorance we can do little better than guess at the value of this variable. If we are going to guess at one of the stages of our calculation, we might just as well guess at the final result to start with, and thus save ourselves many hours of hard work. Furthermore, there are instruments available which will tell us without guesswork what the amplitudes during critical speeds are, and if our calculations do not agree with the instruments, we will have to believe the experimental data anyway, until we know more about

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

the subject. When sufficient consistent experimental data have been obtained, it will no doubt be possible to estimate within very close limits what the damping factor of a particular engine type will be, and using this value, we will be able to predict before building any engine very closely what the vibration characteristics will be. Until such data are available, however, we will have to content ourselves with being able to predict the location of the critical speeds and will have to use our judgment, based on experience in estimating the seriousness of any particular critical speed.

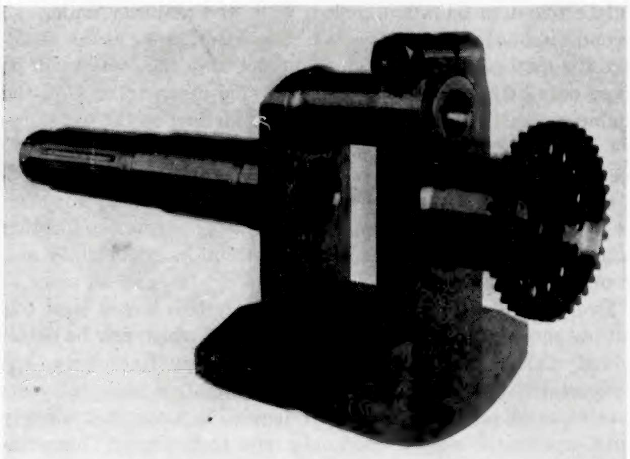

Fig. 1 Single-Throw Crankshaft

We will consider radial engines first, since they are the most orthodox in their behavior where torsional vibration is concerned. Strangely enough, radial engines show more violent vibration amplitudes during a critical speed than do line engines. Vibration torques reach almost unbelievable magnitudes in the case of some of the large engines. We will try to discover the reason for this as we proceed.

Most successful radial engines of today are single-row types having single-throw crankshafts. Following in general our proposed line of attack, we will first see how the natural frequencies work out.

Fig. 1 shows a conventional single-throw crankshaft as used in Wright engines.

The natural vibration frequencies of single-row radials with such shafts will usually be found to lie between 150 and 300 cycles per second with the small displacement engines on the high-frequency end of the list. The author has found that over the conventional range of displacements the natural frequency of similar engines varies inversely as the displacement raised to the 0.30 power quite closely, or

$$\frac{n_1}{n_2} = \left(\frac{D_2}{D_1}\right)^{0.30}$$

Accepting this formula as a rough approximation, we can make a fair guess at the range of natural frequencies which may be expected with present-day single-row radials. The main difference between engines of the same displacement will be the torsional stiffness of the crankshaft. If we assume certain extreme proportions for the dimensions of possible shafts, we can locate approximately the limiting curves as shown by Fig. 2. Curve *A–B* will pertain to engines having relatively stiff crankshafts (usually short nosed), while curve *C–D* will pertain to engines having relatively very flexible shafts (usually long-nosed). These curves are of interest since they facilitate the study of any particular family of engines—particularly so in the case where the design of a whole series of engines is contemplated. In such an instance the proportions of the basic design can be so determined as to preclude the coincidence of a critical speed with the operating range of any one of the models of the series.

Concerning two-row radials, few data are available. In general, however, two-row radials will be found to have higher natural frequencies than single-row engines of the same displacement. Two-row frequencies are indicated by the estimated curve *E–F* on Fig. 2. The author wishes to point out here that these curves are empirical and must be used only with considerable discretion.

After determining the natural-vibration frequency of the crankshaft assembly of an engine, the next step is to find what harmonics of the torque curve will be cumulative for a particular engine. In the case of radial engines, we discover that as we combine the successive harmonics of the torque curves of the cylinders in their firing order, all harmonics cancel out with the exception of those which are multiples of the number of cylinders. It is customary when making a harmonic analysis to assume that all connecting rods center on the crankpin. Such an assumption is not true, of course, when a master rod with articulated rods is used. Link-rod motion modifies the torque curve somewhat, the net result being the introduction of a second harmonic torque variation of small magnitude. This effect may be neglected when making an analysis, since the critical speed for a second harmonic is usually far beyond the normal range of the engine. From the preceding, then, we may make a tabulation of the engine-torque components which may give trouble. (See Table 1.)

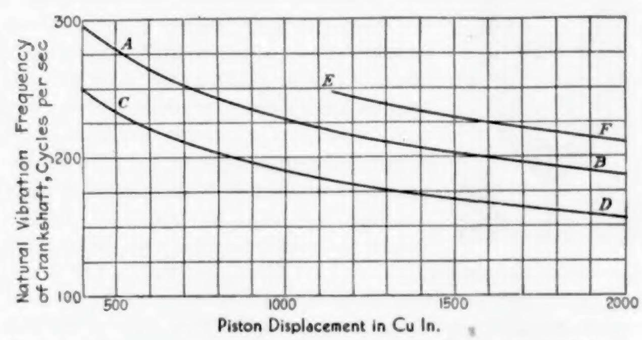

Fig. 2 Natural Vibration Frequency of Crankshafts Versus Piston Displacement, Radial Engines

TABLE 1

Engine types	Critical harmonics
5-cylinder radial, single-row	2, 5, 10, 15, 20, etc.
7-cylinder radial, single-row	2, 7, 14, 28, etc.
9-cylinder radial, single-row	2, 9, 18, 27, etc.
11-cylinder radial, single-row	2, 11, 22, 33, etc.
14-cylinder radial, two-row	2, 14, 28, etc.

Knowing which harmonics are effective, we locate the critical speeds as follows:

$$N = \frac{n(120)}{K}$$

where N = critical speed in rpm

n = natural vibration frequency of crankshaft (cycles per second)

K = number of harmonic torque component, as per Table 1, known as harmonic coefficient.

Let us assume that we have a whole series of single-row engines whose natural frequencies lie along curve *C–D*. We will further assume that the displacements of our models are as given in Table 2.

TABLE 2

	Cu in.	Rated speed (rpm)
5-cylinder model	500	2000–2400
7-cylinder model	700	2000–2400
1st 9-cylinder model	900	2000–2400
2d 9-cylinder model	1800	1800–2200

Referring now to our frequency curve and to Table 1, we can assemble the data for Table 3.

TABLE 3

Engine model	Natural frequency of crankshaft	Critical harmonics and speeds (rpm): Harmonic	Critical speed
5-cylinder	232	5	5570
		10	2320
		15	1855
7-cylinder	211	7	3600
		14	1800
1st 9-cylinder	196	9	2610
		18	1305
		9	2145
2d 9-cylinder	161	18	1072

We notice from this tabulation that the only criticals which fall in our proposed operating range are the 9th harmonic of the nine-cylinder engine and the 10th harmonic of the five-cylinder engine. Reference to a harmonic analysis will show that the magnitudes of these two harmonics are nearly the same, and both may give trouble. Torsiometer tests verify these deductions. In addition to the harmonics which fall directly in the operating range, other harmonics may also give trouble. If a large harmonic lies just below the rated operating range, it will probably give trouble at cruising speeds. Similarly, a critical speed above the rated rpm may give trouble at diving speeds in the case of pursuit ships or diving bombers.

The torsiograms of Fig. 3 were made by a Prescott torsiometer.

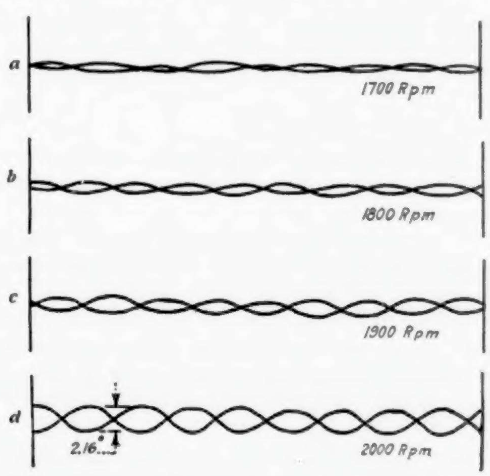

FIG. 3 TORSIOGRAMS MADE BY 9-CYLINDER SINGLE-ROW RADIAL ENGINE OF 1820 CU IN. DISPLACEMENT

(See paper, "Vibration Characteristics of Aircraft-Engine Crankshafts," by Ford L. Prescott, presented at the Fourth National Aeronautic Meeting of the A.S.M.E., paper AER-52-19.) They show the reaction of large nine-cylinder radial direct-drive engine of 1820 cu in. displacement to its critical harmonic (9th). The torque variation of this harmonic was approximately ± 9000 lb-in. The calculated critical speed for this engine was approximately 2200 rpm. The instrument was driven directly from the free end of the crankshaft. A vertical displacement of $^1/_8$ in. represents 1 deg of torsional deflection at the crankshaft. The length of the card represents one revolution of the crankshaft. It will be noticed that two curves are recorded; these curves are two supplementary revolutions of the crankshaft, and represent a complete cycle.

From these torsiograms we can plot the curves of Fig. 4 showing the vibration torque and deflection of the crankshaft during resonant vibration. The curves represent conditions at 2000 rpm, as shown by corresponding torsiograms.

Notice from these curves the tremendous torque variation which apparently occurs during resonant vibration. Remembering that the disturbing harmonic in this case had an amplitude of 9000 lb-in., the ratio of torque variation at resonance to normal torque variation is 128000/9000, or 14.2 to 1. The damping factor (Δ) becomes 1/14.2, or 0.070.[2]

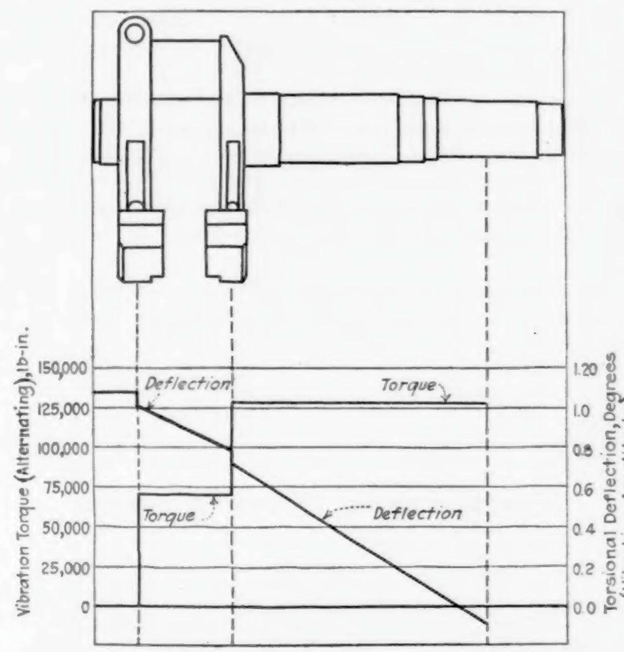

FIG. 4 TORQUE AND DEFLECTION CURVES FOR RESONANT SPEED

Major B. C. Carter states in his paper, "Dynamic Forces in Aircraft Engines," that the damping in radial engines appears to be of such an order that the torque variations in the shaft at synchronous speeds are from 5 to 10 times the normal gas-torque variations. The damping factor corresponding to this would vary between 0.1 and 0.2. It appears from the results presented herewith that the damping is less than what Major Carter suggests. The torsiogram from which our damping factor is calculated does not quite represent synchronous vibration either, being taken at a speed approximately 175 rpm below the critical speed of the engine.

Referring again to the torque curve, note the magnitude of the torque applied at the rear end of the crankpin. Mathematical analysis of the forces acting on a three-bearing crankshaft such as the one under consideration shows that approximately 38 per cent of engine torque is applied to the crankpin during normal operation. In the case of this engine the mean torque carried amounted to approximately 8300 lb-in. Due to resonant vibration we have an additional alternating torque of ± 70,000 lb-in. applied at the rear of the crankpin. The need for a good joint between the rear cheek and the crankpin is im-

[2] The resonance factor is:

$$\frac{1}{\sqrt{(1 - X^2)^2 + \Delta^2 X^2}}$$

where X is the ratio of the disturbing force to the natural frequency of the system and Δ is the damping factor. At resonance $X = 1$. The resonance factor becomes $1/\Delta$. Therefore $\Delta = 1 \div$ resonance factor.

mediately apparent. A well-designed clamp joint at this point will carry as much as 90,000 lb-in. torque.

It is difficult to say offhand just where the maximum actual stress occurs in a crankshaft during resonant vibration. The highest calculated stress will usually be found to occur in the section under the rear propeller cone. Oil holes and keyways, however, produce stress concentrations at other points of the shaft, and highest actual stress will usually be found in some section that is drilled or slotted. Fatigue cracks may therefore occur anywhere from the rear main journal to the propeller splines, but almost invariably do they start at some hole or sharp corner. Ample fillets, smooth holes with well-rounded edges, and careful machining operations are safety measures which must be adhered to faithfully.

We note in our fictitious analysis that the 10th harmonic of the torque curve would come into resonance in the assumed operating range of the five-cylinder engine, and we may expect trouble. This deduction is not idle speculation. Actual vibration of a five-cylinder engine crankshaft in resonance with a 10th harmonic torque variation has been recorded by torsiometer tests.

One more point should be mentioned here before we leave radial engines. We have noted that, neglecting link-rod action, the lowest harmonic of the torque curve which may cause trouble is that one whose coefficient corresponds to the number of cylinders of the engine. Furthermore, reference to a sample harmonic series will show us that the magnitude of the harmonic components decreases rapidly as the higher frequency harmonics are reached. Thus the 12th harmonic is, approximately, 25 per cent of the 6th, or again the 12th harmonic is only 45 per cent, approximately, of the 9th. If we now assume the damping factor to be constant, we know that the vibration amplitudes will be roughly proportional to the amplitude of the forcing torque. The best method of controlling vibration therefore is to use a large number of cylinders. For this reason two-row radials of 14 or more cylinders are particularly attractive. Such engines can usually be operated directly in a critical range without any observable damage.

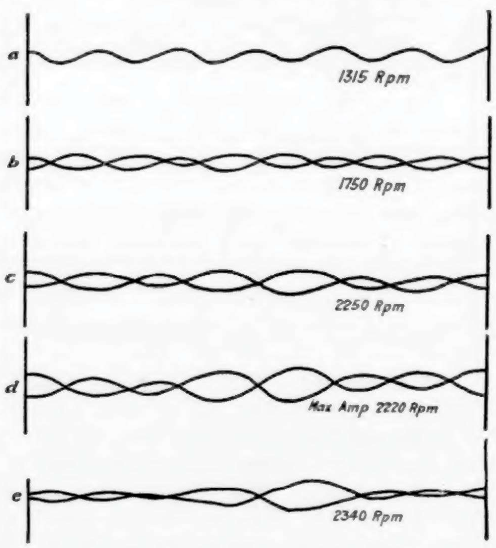

Fig. 5 Torsiograms Made by a 12-Cylinder Inverted-Vee Engine of 1560 Cu In. Displacement

The vibration characteristics of line engines are somewhat different from those of radial engines. The calculation of natural frequencies may be done in the same way as were those for single-throw shafts. It is when we attempt to determine which harmonics of the torque curve are effective that we come across some apparent discrepancies. Some actual examples will illustrate this.

The torsiogram of Fig. 5*a* is a transcript of a card made by a 12-cylinder inverted air-cooled vee engine of 1560 cu in. displacement. The card shows six cycles per revolution, or 12 in two revolutions. In other words, it represents the critical speeds for the 12th harmonic of the torque curve. If we make a harmonic analysis for such an engine as this, and combine the successive harmonics of the single-cylinder torques of the cylinders in their firing order, we discover as before that so far as resultant torque on the whole crankshaft is concerned, all harmonics cancel out except those which are multiples of the number of cylinders. One writer has suggested that since in a line engine we always have pairs of crank throws whose torque curves are exactly 360 deg out of phase, we may combine the turning-effort curve of each pair and let our cycle of events be 360 deg of the crankshaft instead of 720 deg. Doing this automatically eliminates all half-order criticals, which is a mistake, as will be seen.

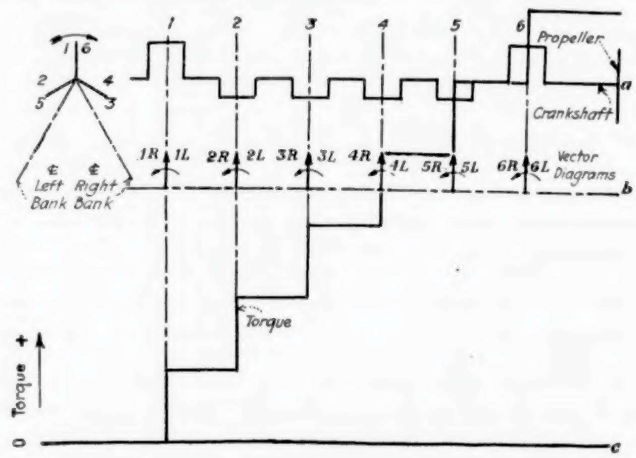

Fig. 6 Summation of 12th Harmonic Torque Variation of 12-Cylinder Inverted-Vee Engine

Referring to torsiograms of Fig. 5*b* and Fig. 5*c*, both made by the same engine that made Fig. 5*a*, we find that the 9th and also the 7th harmonic of the torque curve cause resonant vibration. Our previous analysis would indicate that these harmonics should cancel out. The reason for this apparent discrepancy lies in the displacement of the points of application of the successive torque components. A simple graphical representation will show this.

Fig. 6*a* shows a line diagram of a six-throw crankshaft as used in a 12-cylinder vee engine. Assume that we have an inverted engine as just described, with the banks designated as right and left when viewed from behind the engine, then cylinders 1 right and 1 left are to the rear. The conventional firing order for this engine is: 1R, 6L, 5R, 2L, 3R, 4L, 6R, 1L, 2R, 5L, 4R, 3L.

We can now draw vector diagrams showing the phase relation of the torque components of each cylinder, considering each of the harmonics of the torque curve in order. Fig. 6*b* shows such a vector diagram for the 12th harmonic (harmonic torque of No. 1 right assumed maximum). Fig. 6*c* shows the summation of applied torque plotted against the points of application along the crankshaft. Notice that the harmonic components of the cylinders add up directly from No. 1 to No. 6 throw. Figs. 7 to 17, inclusive, show similar diagrams for each of the other first eleven harmonics of the torque curve. Notice particularly that on some of these diagrams (Fig. 9 for instance) there is a resultant average torque applied to the crankshaft between No. 1 and No. 6 throws, although the sum of the torque is zero at No. 6 throw. In a case such as this, therefore, where No. 6 throw is very near the node, we are probably safe in concluding that the crankshaft will respond to the average torque applied along its length rather than to the algebraic sum of the torque over the whole length. Our diagrams indicate that the 12th, 9th, 7th, 5th, and 3d harmonics will produce an average torque as described, while the other harmonics show none at all or at least only a very small average torque from No. 1 and No. 6 throw. Notice that the forcing torques of the 6th harmonic cancel out at every throw. This analysis agrees with the known results as indicated by tests on this type of engine. No torsiograms of a 5th or 3d harmonic have been recorded to the author's knowledge, but such vibration probably would be encountered were it possible to attain high enough speeds to come into resonance with these harmonics. By such an analysis as this we may study the effect of various crank arrangements, firing orders, and the effect of changing the angle between banks.

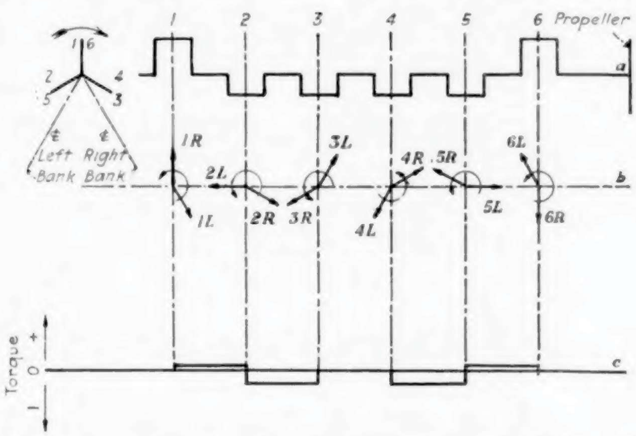

FIG. 7 SUMMATION OF 11TH HARMONIC TORQUE VARIATION OF 12-CYLINDER INVERTED-VEE ENGINE

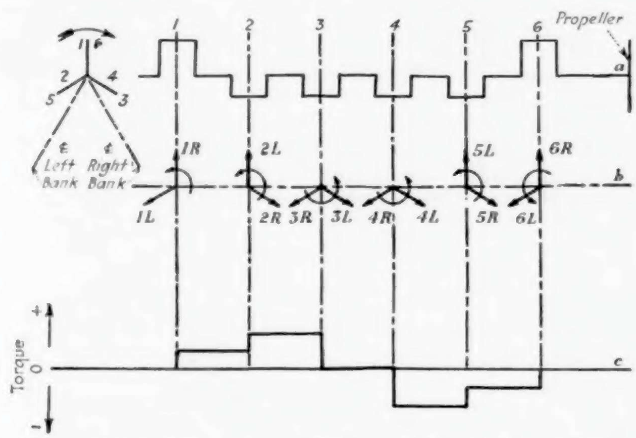

FIG. 10 SUMMATION OF 8TH HARMONIC TORQUE VARIATION OF 12-CYLINDER INVERTED-VEE ENGINE

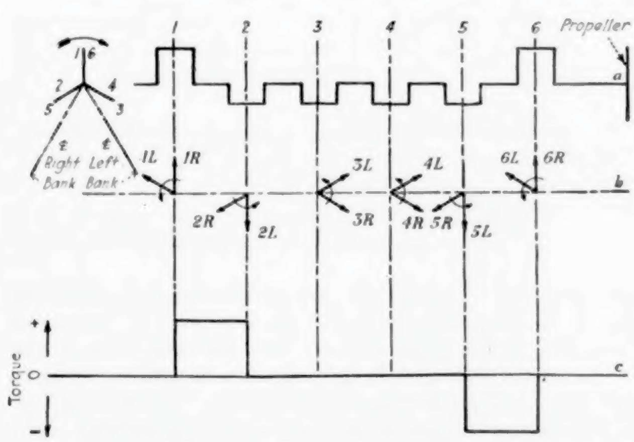

FIG. 8 SUMMATION OF 10TH HARMONIC TORQUE VARIATION OF 12-CYLINDER INVERTED-VEE ENGINE

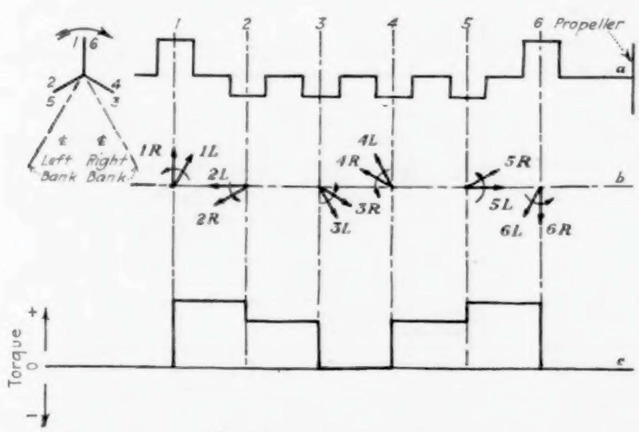

FIG. 11 SUMMATION OF 7TH HARMONIC TORQUE VARIATION OF 12-CYLINDER INVERTED-VEE ENGINE

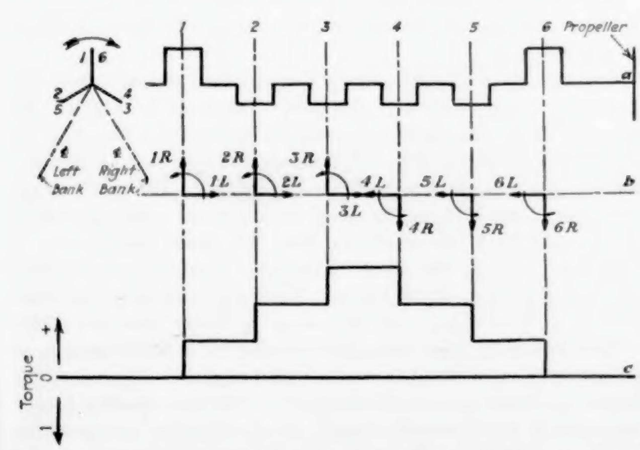

FIG. 9 SUMMATION OF 9TH HARMONIC TORQUE VARIATION OF 12-CYLINDER INVERTED-VEE ENGINE

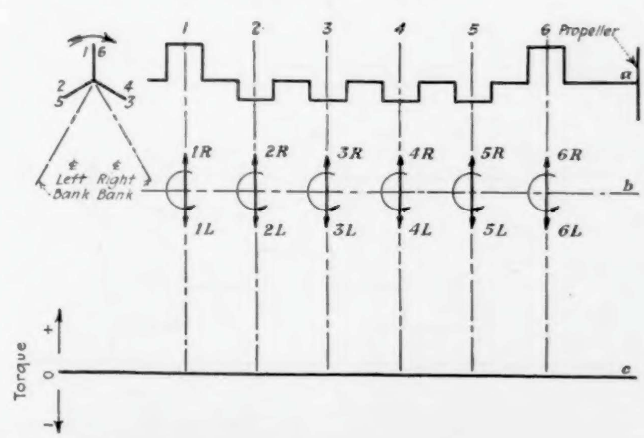

FIG. 12 SUMMATION OF 6TH HARMONIC TORQUE VARIATION OF 12-CYLINDER INVERTED-VEE ENGINE

Referring again to the torsiograms, notice that Fig. 5c and Fig. 5d show a peculiar form having large and small waves. Before we can estimate the crankshaft stresses we must determine which portion of the deflection recorded thus is to be attributed to vibration and which to normal asynchronous torque variation. Since the power impulses in a line engine are not applied to one point of the shaft but to successively different points, the free end of the shaft will not have a steady deflection due to mean torque, but will exhibit a more complicated motion.

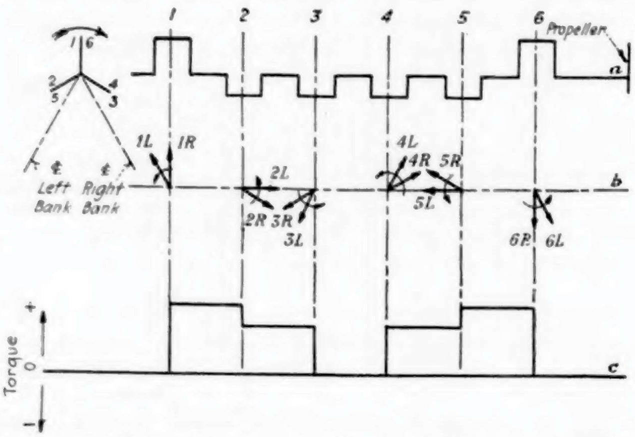

Fig. 13 Summation of 5th Harmonic Torque Variation of 12-Cylinder Inverted-Vee Engine

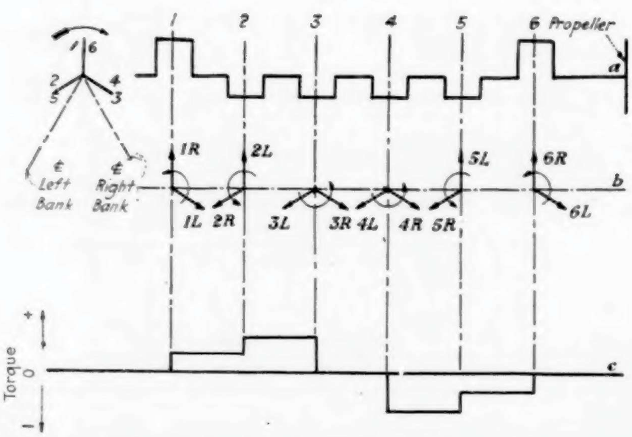

Fig. 14 Summation of 4th Harmonic Torque Variation of 12-Cylinder Vee Engine

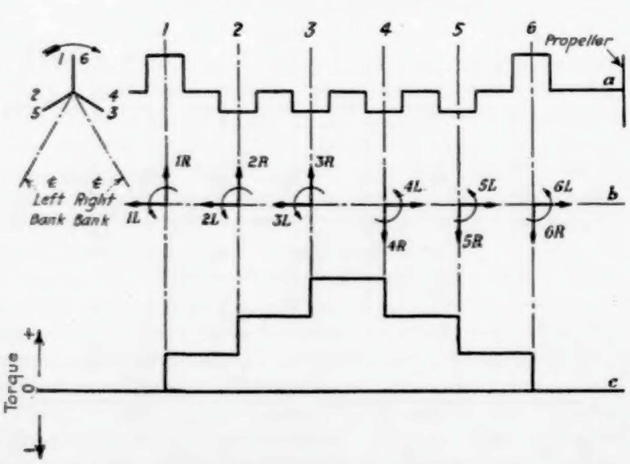

Fig. 15 Summation of 3d Harmonic Torque Variation of 12-Cylinder Vee Engine

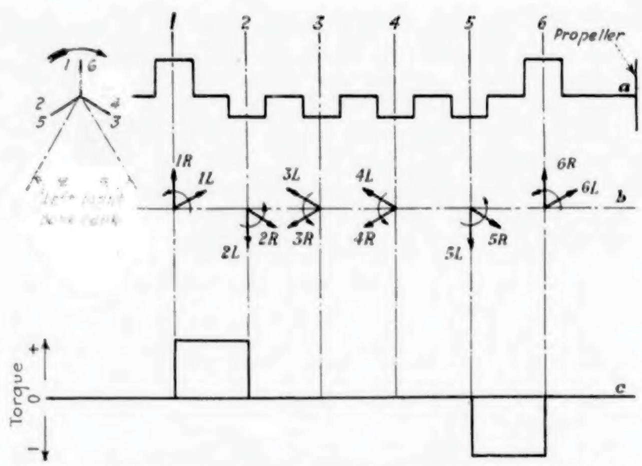

Fig. 16 Summation of 2d Harmonic Torque Variation of 12-Cylinder Vee Engine

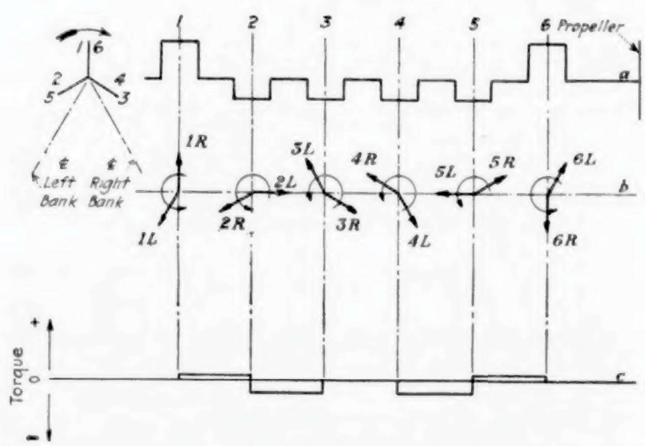

Fig. 17 Summation of 1st Harmonic Torque Variation of 12-Cylinder Inverted-Vee Engine

During a critical speed there will be superimposed upon this asynchronous motion a harmonic vibration, and the torsiometer will record the resultant of these two vibrations. The easiest way to differentiate between these curves is to draw the enveloping lines of the higher frequency vibration. Fig. 18 is an example. Notice that the curve described by the envelope corresponds to the torsiogram which recorded the motion of the free end of the crankshaft at an asynchronous speed (see Fig. 18d).

Fig. 19 shows the elastic curve and the torque curve of a 12-cylinder engine crankshaft during resonant vibration, as shown by Fig. 5d.

Since we do not know exactly the resultant magnitude of the disturbing force of the 7th harmonic indicated by Fig. 5d, we are safer if we calculate our damping factor from the diagram of the 12th harmonic (Fig. 5a), because we know that all cylinder components add up directly (refer to Fig. 6c). The total torque variation of the 12th harmonic of the engine at 1315 rpm based on propeller-load characteristics from 600 brake horsepower at 2400 rpm was ± 620 lb-in. The total torque variation indicated by Fig. 5a is ± 9500 lb-in. The resonance factor becomes 9500 ÷ 620, or 15.3, and the damping factor becomes 0.065.

This would indicate that the damping in a line engine is of about the same order as that in a radial engine. The redeeming feature of a line engine in this respect is that the effective torque variation of the harmonic which usually falls in the operating range (the 7th) is small. Compare the torque summation of Fig. 6 and Fig. 11. For this reason it is not unusual to be able to operate directly in the range of this harmonic without serious difficulty, providing that the accessories are driven from the propeller end of the engine or through some kind of flexible coupling.

Should the 12th harmonic of such an engine as described fall in the operating range, considerable trouble could be expected. Reference to Fig. 6 will remind us that the effective torque variation of this harmonic is large, and large vibration amplitudes could be expected. Crankshaft trouble, excessive bearing wear, and valve-gear failures would not be surprising. Fortunately, this harmonic usually lies in the idling range of this type of engine.

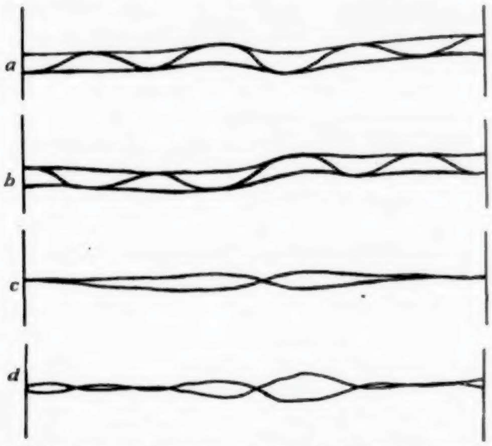

Fig. 18 Method of Distinguishing Between Asynchronous and Resonant Vibration

(*a*, first revolution of Fig. 5*d*; *b*, second revolution of Fig. 5*d*; *c*, combination of enveloping curves of *a* and *b*; *d*, motion at asynchronous speed.)

In contrast with this we have previously noted that the major critical speed of a radial engine often falls in the operating range. That is why crankshaft vibration may be expected to give more trouble in a radial than in a line engine. Experience has taught us that this is true.

No discussion about torsional vibration would be complete without at least some mention of vibration dampers, and to be orthodox we will not deviate from this custom. Vibration dampers occupy more or less the position of poor relations to the aircraft-engine industry. We will welcome them with open arms if and when they come with real value in their hands, but until such a time we prefer not to be bothered with them. There are several objections to using conventional types of dampers in present-day conventional airplane engines. The three main objections are:

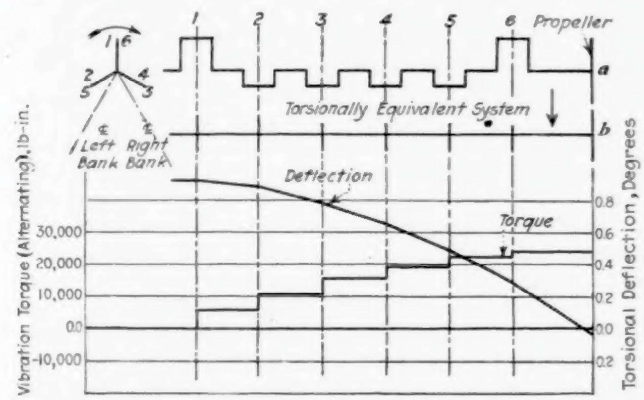

Fig. 19 Torque and Deflection Curves for Resonant Speed, Line Engine

1 They add weight
2 They are, in some forms, susceptible to misadjustment
3 They do not eliminate vibration; they only damp it.

Faced with these and other objections, we prefer to modify our engine designs to avoid resonance rather than to resort to dampers. However, we are open to persuasion, and perhaps some day we will make the addition of a vibration damper a further sales attraction, as do the automobile builders now.