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THERMODYNAMIC DATA FOR THE COMPUTATION OF THE PERFORMANCE OF EXHAUST-GAS TURBINES

By Benjamin Pinkel and L. Richard Turner

Aircraft Engine Research Laboratory
Cleveland, Ohio

NACA

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ADVANCE RESTRICTED REPORT

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SUMMARY

Information published in chemical journals from 1933 to 1939 on the thermodynamic properties of the component gases of exhaust gases based on spectroscopic measurements were used as data for computing the ideal values of work, mass flow, nozzle velocity, power, and temperature change involved in the thermodynamic processes of a gas turbine. Curves from which this information can conveniently be obtained are given. An additional curve is included from which the heat flow may be calculated for nonadiabatic processes.

A method of computation is presented in which the thermodynamic quantities associated with an isentropic process are calculated by the use of two effective values of the ratio of specific heats γ simply related to the value of γ at the start of the process and to the pressure ratio. These values of γ are used in the equations derived on the assumption of constant specific heat and thus permit convenient algebraic manipulation of the thermodynamic quantities. The relation of these values of γ to the conventional thermodynamic functions and the condition for the validity of the method is derived. This method applies accurately for thermodynamic processes occurring within the temperature range of about 700° to 2700° F absolute.

INTRODUCTION

In the computation of turbine efficiency from test data, the power output of a turbine may be determined from dynamometer-stand tests or their equivalent. The power input or the ideal power available from the exhaust gas, however, must be computed from the thermodynamic properties of the gas. Other items of interest in exhaust-gas-turbine computations are the ideal temperature drop, nozzle velocity and mass flow. In these computations various organizations concerned with the testing of turbines have been using tables derived from different sources and involving different assumptions and approximations. This report was prepared at the request of the NACA Subcommittee on Recovery of Power from Exhaust Gas for standardizing the data involved in a computation of turbine efficiency and the other important items of turbine performance.

The thermodynamic properties of the component gases of exhaust gas taken from references 1 to 7 are tabulated for a temperature range from 540° to 2700° F absolute, and equations and tables are given for computing these properties for exhaust gas for any given fuel-air ratio of the mixture and hydrogen-carbon ratio of the fuel. The basic data were originally computed from spectroscopic measurements, which are at the present time considered to be the most accurate source of information on the thermodynamic properties of gases at high temperatures. In order to lessen the labor on the part of the user, curves of the ideal work, mass flow, nozzle velocity, and temperature drop covering the range of fuel-air ratios from 0 to 0.12, hydrogen-carbon ratios from 0.084 to 0.200, initial gas temperatures from 1200° to 2400° F absolute, and pressure ratios from 1 to 10 are given.

This analysis was completed at the Aircraft Engine Research Laboratory of the National Advisory Committee for Aeronautics, Cleveland, Ohio, in August 1943.

SYMBOLS

A area, (sq ft)

c_m mass coefficient of discharge, (lb)/(theoretical lb)

C_p specific heat at constant pressure, (Btu)/(lb mole)(°F)

c_p specific heat at constant pressure, (Btu)/(lb)(°F)

c_v specific heat at constant volume, (Btu)/(lb)(°F)

E_0 the energy zero, or the energy of combustion at the absolute zero of temperature, (Btu)/(lb mole)

F Gibbs' free energy

f fuel-air ratio, (lb)/(lb)

g 32.2 (lb)/(slug)

H enthalpy, (Btu)/(lb mole)

h enthalpy, (Btu)/(lb)

J mechanical equivalent of heat, 778 (ft-lb)/(Btu)

K equilibrium constant

K_R correction factor for changes in mean molecular weight

K_γ correction for changes in mean molecular specific heat

K_μ combined correction to the mass flow per unit area

M mass flow of fluid, (slug)/(sec)

m hydrogen-carbon ratio of the fuel, (lb)/(lb) (assumed atomic weights, hydrogen 1.008, carbon 12.01)

M_a mean molecular weight of air, (lb)/(lb mole)

P power, (hp) or (ft-lb)/(sec)

P_s turbine shaft power, (hp) or (ft-lb)/(sec)

p pressure, (lb)/(sq ft)

q heat quantity added to a fluid, (Btu)/(lb)

R universal gas constant, 1545.7 (ft-lb)/(lb mole)(°F)

R_a gas constant for air, (ft-lb)/(lb)(°F)

R_b gas constant for a gas mixture, (ft-lb)/(lb)(°F)

S entropy of the ideal gas at 1 atmosphere, (Btu)/(lb mole)(°F)

s entropy of the ideal gas at 1 atmosphere, (Btu)/(lb)(°F)

T temperature, (°F absolute)

u velocity, (ft)/(sec)

v volume, (cu ft)

W work done by a gas, (ft-lb)/(lb)

W_{th} ideal work in thermodynamic process, (ft-lb)/(lb)

γ ratio of the specific heats of a fluid

γ_h effective value of γ for enthalpy-change computations

γ_t effective value of γ for temperature-change computations

η_t turbine-shaft efficiency

ρ density, (slug)/(cu ft)

Subscripts:

1 refers to conditions at higher pressure or temperature

2 refers to conditions at lower pressure or temperature

a air

b burned mixture

cr critical

ANALYSIS AND DISCUSSION

Simplified Method of Thermodynamic Computation

Ideal turbine power available. - If the heat transfer to the surrounding medium is neglected, the equation for the conservation of energy gives the following relation between the energy at the entrance and exit of the turbine and the work W done by the gas per unit weight:

$$J\int_0^{T_1} c_p dT + \frac{1}{2g} u_1^2 = J\int_0^{T_2} c_p dT + \frac{1}{2g} u_2^2 + W \qquad (1)$$

The quantity $\int_0^{T} c_p dT$ is called the enthalpy, or heat content, and is usually designated h. For an ideal gas having a constant specific heat, equation (1) reduces to

$$Jc_pT_1 + \frac{1}{2g} u_1^2 = Jc_pT_2 + \frac{1}{2g} u_2^2 + W \qquad (2)$$

If it is assumed that the specific heat in equation (1) does not vary appreciably from its initial value during a given expansion process and that the process is isentropic, the temperature and pressure are connected by the relation

$$T_2/T_1 = (p_2/p_1)^{\frac{\gamma-1}{\gamma}}$$

and equation (1) becomes

$$R_bT_1 \frac{\gamma}{\gamma-1}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right] + \frac{1}{2g}\,u_1^{\,2} - \frac{1}{2g}\,u_2^{\,2} = W \qquad (3)$$

When the approach velocity u_1 is small as is often the case, $\frac{1}{2}\,u_1^{\,2}$ may be neglected. Since a turbine or other working device can theoretically be designed to have zero leaving velocity u_2, the ideal work W_{th} that may be derived from the gas in a flow process on expansion between the pressures p_1 and p_2 is given by

$$\frac{W_{th}}{R_bT_1} = \frac{\gamma}{\gamma-1}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right] \qquad (4)$$

Where the approach velocity u_1 is large, the term $u_1^{\,2}/2gR_bT_1$ should be added to the right-hand side of equation (4) to obtain the total ideal work available. An alternative and possibly more convenient method of taking care of u_1 is to use the stagnation temperature and total pressure in equation (4) for T_1 and p_1, respectively.

In the case of an actual gas the assumption made in the derivation of equations (2), (3), and (4) that the specific heat does not vary during the expansion process is not strictly correct. The fundamental method of computing W_{th}/R_bT_1 that takes into account the variation in specific heat during the expansion process is given in detail in appendixes A and B, together with the tables necessary for computing this quantity for a range of hydrogen-carbon ratios, air-fuel ratios, initial temperatures, and pressure ratios. This method will be called the classical process. It involves the computation of enthalpy and entropy. The data used in these computations and listed in table I were obtained from references 1 to 7 and are based on spectroscopic measurements. The assumptions made in these computations are listed in appendix A.

An alternative procedure, which led to a convenient presentation of this information and a simplified method of computation, is as follows: The value of W_{th}/R_bT_1 was computed by the above-mentioned classical process for a given set of operating conditions (pressure ratio, initial temperature, and exhaust-gas composition). An effective value of γ, designated γ_h, was then computed from this value of W_{th}/R_bT_1 and pressure ratio by means of equation (4). This value of γ_h provides a means of calculating the value of W_{th}/R_bT_1 from the equation

$$\frac{W_{th}}{R_b T_1} = \frac{\gamma_h}{\gamma_h - 1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma_h - 1}{\gamma_h}}\right] \qquad (5)$$

for the specific conditions for which this value of γ_h applies. Values of γ_h computed by this procedure for a range of conditions bracketing the operating conditions of interest in exhaust-gas-turbine applications are shown in figure 1 plotted against γ_1 for the pressure ratios p_1/p_2 of 4, 6, and 10 for a range of temperatures and for several mixtures, namely,

Constituents	Fuel-air ratio
Air	0
Air + octane	.0662
Air + octane	.100
Air + benzene	.100

When γ_h is divided by γ_1, all the data similar to that in figure 1 can be plotted on a single curve against pressure ratio as shown in figure 2(a). Thus in the range of gas-turbine applications, the value of γ_h can be obtained from the value of γ_1 and pressure ratio by means of figure 2(a).

The decrease in scatter of the points about the faired curves in figure 1 with increase in pressure ratio is noted. The characteristics of equation (4) are such that small inaccuracies in the value of $W_{th}/R_b T_1$ introduce relatively large dispersions in the value of γ_h calculated from equation (4) for pressure ratios p_1/p_2 near unity; the dispersion decreases as pressure ratio is increased. Thus small irregularities in the tabulated values of entropy and enthalpy as, for example, a variation of one unit in the third decimal place of entropy, cause considerable scatter in the relation between γ_h and γ_1 for the lower pressure ratios. The decrease in scatter as the pressure ratio is increased demonstrates the fundamental soundness of this method, which is in effect a method of fairing specific-heat data.

Because γ_h/γ_1 is a function only of pressure ratio in the present case, it is apparent from equation (5) that $W_{th}/R_b T_1$ is a function of γ_1 and the pressure ratio. Figure 3 is a plot of $W_{th}/R_b T_1$ against pressure ratio p_1/p_2 and γ_1 obtained by means of equation (5) and figure 2(a).

The instantaneous values of γ are shown in figure 4 plotted against the fuel-air ratio and the temperature for two values of hydrogen-carbon ratio. The spread of the curves with hydrogen-carbon ratio is small, and linear interpolation between the two values given will yield accurate results.

The value of the gas constant R_b is given in figure 5 plotted against fuel-air ratio and hydrogen-carbon ratio. In the figures and tables shown, air was assumed to be dry with the composition

N_2 percent by volume 78

O_2 percent by volume 21

A percent by volume 1

which has a mean molecular weight of 28.97 (lb)/(lb mole) and a gas constant R_a of 53.35 (ft-lb)/(lb)(°F). The method of computing R_b and γ_1 is described in detail in appendix B.

Ideal temperature drop. - For the case in which the specific heats are constant, the temperature ratio T_2/T_1 in an isentropic process is related to the pressure ratio as follows:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \tag{6}$$

This relation does not apply in the actual case in which the specific heats vary during the thermodynamic process. The procedure previously described can, however, be applied to this case. The temperature ratio for any given set of conditions is computed by the classical process from the data given in table II. An effective value of γ for temperature computations, designated γ_t, is then computed from equation (6) and the known values of temperature ratio and pressure ratio. The values of γ_t were computed over the same range of temperatures, pressure ratios, and gas compositions used in the computation of γ_h. As in the case of γ_h it was found that the ratio of γ_t to γ_1 was a function only of pressure ratio in this range of conditions. The ratio of the value of γ_t to γ_1 is shown in figure 2(b) plotted against pressure ratio. Thus the temperature ratio in an isentropic process can be computed from the equation

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma_t-1}{\gamma_t}} \tag{7}$$

and the data given in figure 2(b). It is apparent from equation (7) and figure 2(b) that T_2/T_1 may be presented as a function of p_1/p_2, γ_1. A plot of this function is shown in figure 6.

Figure 7 shows a plot of $-J\Delta h/R_bT_1$ against T_2/T_1 and γ_1 obtained from figures 3 and 6. Although figures 3 and 6 relate only to isentropic processes, figure 7 is not so restricted because $J\Delta h/R_bT_1$ as a function of temperature change is independent of the type of process. Figure 7 may, therefore, be used to compute changes in enthalpy arising from any cause, such as heat addition or removal by heat transfer or other nonisentropic processes. In isentropic processes $-J\Delta h$ is equal to W_{th}.

Ideal density ratio. - The equation for the density ratio ρ_2/ρ_1 follows from equation (7) and the gas law

$$\frac{\rho_2}{\rho_1} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma_t}} \tag{8}$$

Ideal nozzle velocity. - The ideal nozzle velocity may be obtained by equating the kinetic energy at the nozzle to the theoretical work

$$\frac{1}{2} u_2^2 = gW_{th}$$

from which

$$u_2 = \sqrt{2gW_{th}} \tag{9}$$

Ideal mass flow. - The ideal mass flow is given by $M = \rho_2 u_2 A$. From equations (5), (8), (9), and the perfect gas law

$$\frac{M\sqrt{gR_bT_1}}{p_1A} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma_t}} \sqrt{\frac{2\gamma_h}{\gamma_h - 1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma_h - 1}{\gamma_h}}\right]} \tag{10}$$

This relation holds for a convergent-type nozzle for subsonic velocities and for convergent divergent nozzles of the proper shape over the entire flow range.

For flow at a greater-than-critical pressure ratio through a convergent nozzle, the mass flow has the critical value. The mass flow at critical pressure ratio has been computed as a function of γ_1, assuming that critical flow exists when the local Mach number is unity at the nozzle throat. For this calculation it was necessary to know the instantaneous value of γ_2 at the throat. The ratio γ_2 divided by γ_1 was computed and found to be very nearly a function of pressure ratio only. From these data the quantity

$$\frac{M_{cr}\sqrt{gR_bT_1}}{p_1A}$$

has been computed. The results are shown in figure 8 plotted against γ_1. The critical pressure ratio is also shown in this figure plotted against γ_1.

Theoretical basis for effective values of γ. - The conditions for which the foregoing presentation involving the use of effective values of γ is accurate are derived from theoretical considerations in appendix C. It is shown that in the range in which log γ plotted against Js/R_b is a straight line, the following relations are obtained for isentropic processes:

1. γ_2/γ_1 is a function only of p_1/p_2.

2. γ_t/γ_1 is a function only of p_1/p_2.

3. γ_h/γ_1 is a function of γ_1 and p_1/p_2; however, for the range of conditions of present interest, its dependence on γ_1 is negligible.

4. T_2/T_1 is a function only of γ_1 and p_1/p_2.

5. $\Delta h/R_bT_1$ is a function only of γ_1 and p_1/p_2. The following equations are derived. (See equations (44) to (46) of appendix C.)

$$\frac{\gamma_2}{\gamma_1} = \left(\frac{p_1}{p_2}\right)^{-r} \tag{44}$$

$$\frac{\gamma_t}{\gamma_1} = \left(\frac{p_1}{p_2}\right)^{-\frac{r}{2}} \tag{45}$$

$$\frac{\gamma_h}{\gamma_1} = \left(\frac{p_1}{p_2}\right)^{-\frac{r}{3}} \tag{46}$$

where r is the slope of the curve of $\log \gamma$ against Js/R_b. Curves are given which show that in the range of gas-turbine application the value of r is -0.014 for the following mixtures:

Constituents	Fuel-air ratio
Air	0
Air + octane	.0662
Air + octane	.100
Air + benzene	.100

The same value of r may be expected to hold for intermediate gas compositions because the same value was found to hold for all the component gases except carbon dioxide. The range of temperatures over which the relation is valid is nearly constant for all the diatomic molecules considered. These molecules are the chief constituents of exhaust gas. In the derivation of the expressions for γ_t/γ_1 and γ_h/γ_1 given, use was made of the fact that r is small compared with unity. The general relations, not limited by this condition, are given in appendix C.

This analysis provides a convenient means of determining the range of validity of the method. Examination of the curves of $\log \gamma$ against Js/R_b reveals that the values of γ for a specified value of s given by a straight line having a slope r differs from the actual value of γ by 0.1 percent or less in the temperature range from 900° to 2500° F absolute. The error in the values of theoretical work or temperature computed from equations (5) and (7) will be less than 0.1 percent for an error of the effective values of γ of 0.1 percent. The method can be used with very good accuracy for thermodynamic process occurring within a temperature range from 700° to 2700° F absolute. This temperature range covers the range of interest in gas-turbine work.

Equations (44) to (46) permit computation for an isentropic process of the temperature corresponding to the higher pressure (subscript 1) and the ideal work when the temperature at the lower pressure (subscript 2) is known. For example, the value of γ_2 corresponding to T_2 can be obtained from figure 4. The quantities γ_1, γ_t, and γ_h can then be computed from equations (44) to (46). The temperature T_1 and ideal work can then be obtained from equations (6) and (5), respectively, and the effective values of γ or from figures 6 and 3 and the value of γ_1.

Working charts for gas-turbine computations. - In figures 9, 10, 11, and 12 the same thermodynamic quantities are presented in a form that was thought to be more familiar to turbine designers and easier to use. In each case the principal curves apply for air and the correction factors take care of other gas compositions. The thermodynamic property given in any figure is multiplied by all of the corrections appearing on that figure. Figure 9 shows the ideal work plotted against pressure ratio and initial gas temperature. The terms K_γ and K_R are correction factors that depend on the fuel-air ratio and hydrogen-carbon ratio. The values of W_{th} taken from figure 9 are multiplied by these correction factors. Figure 10 shows the ideal jet velocity plotted against pressure ratio and initial gas temperature. The values taken from this figure are to be multiplied by the correction factors $K_\gamma^{1/2}$ and $K_R^{1/2}$ to correct for the exhaust-gas composition. Figure 11 shows the ideal mass flow plotted against pressure ratio for various initial temperatures. These values are to be multiplied by the correction factors K_μ and $K_R^{-\frac{1}{2}}$. It is assumed in this figure that the nozzles are of the convergent type and that the mass flow is constant above the critical pressure ratio. Figure 12 shows the ideal power per square inch of nozzle area per inch of mercury of inlet pressure as a function of initial temperature and pressure ratio. The values given by this figure must be multiplied by the correction factors K_μ, K_γ, and $K_R^{1/2}$. In figure 12 the mass flow is taken as the critical value for all pressure ratios above the critical ratio, but the work per pound is taken as the ideal value over the entire pressure-ratio range.

The method by which the correction factors were obtained is described in appendix D.

For the convenience of the reader in preparing enlarged charts, the data from which the curves of this report were plotted are tabulated in tables III to XIII. The correction factors K_γ and K_μ can be computed from table XIII and figure 14 by the use of equations given in appendix D.

Sample Computations

The following computation is given to illustrate the method of obtaining the information from the two sets of curves: (1) figures 3 to 8; (2) figures 9 to 12.

1. Let it be desired to compute the ideal work, power per square inch, temperature drop, mass flow, and velocity for the case of exhaust gas having the fuel-air ratio of 0.090, hydrogen-carbon ratio of 0.189 (octane), initial temperature of 1400° F (1860° F absolute), initial pressure of 30 inches of mercury absolute, final pressure of 10 inches of mercury absolute, and pressure ratio of 3.

A. Ideal work:

From figure 4

$$\gamma_1 = 1.306$$

From figure 5

$$R_b = 57.68$$

From figure 3

$$\frac{W_{th}}{R_b T_1} = 0.9675$$

$$W_{th} = 0.9675 \times 57.68 \times 1860$$

$$= 103,800 \text{ (ft-lb)/(lb)}$$

The value of W_{th} can also be computed from equation (5) and figure 2(a) if greater accuracy than that given by figure 3 is desired.

B. Ideal discharge velocity:

From equation (9)

$$u_2 = \sqrt{2gW_{th}}$$

$$= \sqrt{2 \times 32.2 \times 103,800}$$

$$u_2 = 2585 \text{ (ft)/(sec)}$$

C. Ideal mass flow through convergent nozzle:

From figure 8

$$\frac{M_{cr}\sqrt{gR_bT_1}}{p_1 A} = 0.6693$$

or

$$g\frac{M_{cr}}{A} = 0.1709 \text{ (lb)/(sq in.)(sec)}$$

D. Ideal power:

$$\text{Power per sq in.} = \frac{g}{550} \frac{M_{cr}}{A} W_{th}$$

$$= 32.24 \text{ (hp)/(sq in.)}$$

E. Ideal discharge temperature:

From figure 6

$$T_2/T_1 = 0.768$$

or

$$T_2 = 1428^\circ \text{ F absolute}$$

The value of T_2 could also have been computed by means of equation (7) and figure 2(b).

2. The same information can be obtained from figures 9 to 12.

A. Ideal work:

From figure 9

$$(W_{th})_{air} = 122.3 \text{ (Btu)/(lb)}$$

$$K_R = 1.081$$

$$K_\gamma = 1.0077$$

$$W_{th} = 122.3 \times 1.081 \times 1.0077$$

$$= 133.2 \text{ (Btu)/(lb)}$$

$$= 103{,}600 \text{ (ft-lb)/(lb)}$$

B. Ideal nozzle velocity or ideal discharge velocity:

From figure 10

$$(u_2)_{air} = 2476 \text{ (ft)/(sec)}$$

$$K_R^{1/2} = 1.040$$

$$K_\gamma^{1/2} = 1.0039$$

$$u_2 = 2476 \times 1.040 \times 1.0039$$

$$= 2585 \text{ (ft)/(sec)}$$

C. Ideal mass flow through convergent nozzle:

From figure 11

$$\left(g \frac{M_{cr}}{p_1 A}\right)_{air} = 0.00597 \text{ (lb)/(sq in.)(in. Hg)(sec)}$$

$$K_R^{-1/2} = 0.962$$

$$K_\mu = 0.9932$$

$$g \frac{M_{cr}}{A} = 0.00597 \times 0.962 \times 0.9932 \times 30$$

$$= 0.1711 \text{ (lb)/(sq in.)(sec)}$$

D. Ideal power:

From figure 12

$$\left(\frac{P_{th}}{p_1 A}\right)_{air} = 1.032$$

$$K_\gamma = 1.0077$$

$$K_R^{1/2} = 1.040$$

$$K_\mu = 0.9932$$

$$\text{Power per sq in.} = 1.032 \times 1.0077 \times 1.040 \times 0.9932 \times 30$$

$$= 32.22 \text{ (hp)/(sq in.)}$$

The dotted lines in figures 9 to 12 illustrate the method of reading the values of the correction factors from these figures.

Figures 3, 6, 7, 9, 10, 11, and 12 have been reproduced as large prints suitable for computations. A set of these prints is attached.

CONCLUDING REMARKS

The spectroscopic specific heat data and classical method of computation of thermodynamic properties of gases are given. An alternative method of computation in which the thermodynamic quantities associated with an isentropic expansion are calculated by use of two effective values of ratio of specific heats γ simply related to the value of γ at the start of the process and to the pressure ratio is presented. These values of γ are used in the equations derived on the assumption of constant specific heat and thus permit convenient algebraic manipulation of the thermodynamic quantities.

Two sets of charts for determination of the thermodynamic quantities are given. One set is of a general nature in which nondimensional coefficients are used. In the second set of charts specific data of interest in turbine computations are plotted against turbine operating conditions.

Aircraft Engine Research Laboratory,
National Advisory Committee for Aeronautics
Cleveland, Ohio.

APPENDIX A

LIST OF ASSUMPTIONS

The following assumption are made in computations of this report:

1. The composition of the exhaust gas does not change in going through the thermodynamic process.

2. The composition of the exhaust gas in the mixture range leaner than stoichiometric is based on the condition that the fuel is completely converted to CO_2 and H_2O.

3. The composition of the exhaust gas in the mixture range richer than stoichiometric is governed by the equilibrium equation

$$K = \frac{(CO)(H_2O)}{(H_2)(CO_2)} \tag{11}$$

where the equilibrium constant K is frozen at the value of 3.8. (See reference 8.)

4. The amount of unburned hydrocarbons in the exhaust gas is negligible.

5. The internal energy states of each component gas are in equilibrium.

6. Exhaust gas behaves as a perfect mixture of perfect gases.

With regard to assumption 3 it is known that theoretically the equilibrium constant K depends on the gas temperature. The following values are taken from reference 9:

EQUILIBRIUM CONSTANT FOR
WATER-GAS REACTION

Temperature		K
(°F abs.)	(°C abs.)	
540	300	0.0000103
720	400	.000147
1080	600	.0369
1440	900	.246
1800	1000	.713
2160	1200	1.395
2520	1400	2.20
2880	1600	3.055
3240	1800	3.80
3600	2000	4.56
3960	2200	5.21
4320	2400	5.77
4680	2600	6.22
5040	2800	6.592
5400	3000	6.92

These values are computed from spectroscopic data by means of equations derived by the methods of statistical mechanics. On the other hand, experimental determination of the composition of exhaust gas by D'Alleva and Lovell (reference 8) leads to an average value for the equilibrium constant of 3.8. This value was obtained by analysis of cooled exhaust gas having an initial temperature probably less than 2000° F absolute. At a gas temperature of 2000° F absolute, the table shows a value for K of 1.07; whereas the value for K of 3.8 corresponds to a temperature of 3240° F absolute. The conclusion drawn from this evidence is that the rate of the water-gas reaction is so slow for temperatures below approximately 3240° F absolute that for exhaust-turbine computations the equilibrium may be considered as frozen at the composition corresponding to an equilibrium constant of 3.8. This is also the basis for assumption 1. Although this assumption may be superseded at some later date by a more accurate assumption, it is believed to be considerably more accurate than the assumption that gas is in equilibrium at each temperature in accordance with the table.

APPENDIX B

CLASSICAL THERMODYNAMIC CALCULATION

The method of computing W_{th} is based on the following considerations. The heat added during a thermodynamic process is equal to the sum of the changes in internal energy and work

$$dq = gMc_v dT + \frac{1}{J}\, pdv \tag{12}$$

but

$$pdv = d(pv) - vdp = gMR_b dT - vdp$$

Thus

$$dq = gMc_p dT - \frac{1}{J}\, vdp$$

For the case of zero heat added or subtracted during the expansion process, including heat arising from the formation and dissipation of turbulence,

$$dq = 0 \quad \text{and} \quad gMc_p dT - \frac{v}{J}\, dp = 0 \tag{13}$$

But by the gas law

$$pv = gMR_b T \tag{14}$$

Then

$$\frac{c_p dT}{T} - \frac{R_b}{J}\frac{dp}{p} = 0 \tag{15}$$

$$\frac{R_b}{J} \log \frac{p_y}{p_x} = \int_{T_x}^{T_y} c_p \frac{dT}{T} \tag{16}$$

The quantity $\int_{T_x}^{T_y} c_p \frac{dT}{T}$ is the difference in entropy of the gas at a pressure of 1 atmosphere and is designated by the smybol $\Delta s\ (T) = s(T_y) - s(T_x)$.

Thus

$$\frac{R_b}{J} \log \frac{p_y}{p_x} = s(T_y) - s(T_x) \tag{17}$$

The quantity $s(T)$ for a given gas is a function of T only. The values of $S(T)$, the entropy per mole of the elementary components of exhaust gas obtained from tables in references 1 to 7, are listed in table I. Since the composition of the gas is assumed constant during a given expansion process, the constant entropy of mixing has been neglected in all the calculations.

The ideal work done by the gas during this process is given by equation (1)

$$W_{th} = \int_0^{T_x} c_p dT - \int_0^{T_y} c_p dT \tag{18}$$

where T_x and T_y are the total temperatures. The quantity $\int_0^T c_p dT$ for a given gas in the range of present interest is a function only of T. It is usually designated enthalpy and given the symbol $h(T)$. The values of $H(T)$, or enthalpy per pound mole of the components of exhaust gas given in table I, were taken from references 1 to 7. Thus

$$W_{th} = h(T_x) - h(T_y) \tag{19}$$

The method of computing W_{th}, called here the classical process, consists of the following steps. From the known values of p_x/p_y and T_x the value of T_y for an adiabatic expansion is found from equation (17) and the tabulated values of $S(T)$. Since T_x and T_y are known, the value of W_{th} can be obtained from equation (19) and the tabulated values of $H(T)$.

The values of the thermodynamic functions h, s, c_p, and R are computed on the basis of assumptions given in appendix A. As a result of assumption 6, the heat content of a mixed gas is the sum of the heat content of each component multiplied by the ratio of mass of that component to the total mass of the mixture. A similar relation between the properties of the mixture and those of the constituent gases holds with regard to s, c_p, and R.

In the case of the gas constant R_b the processes may be changed to that of finding the mean molecular weight since the gas constant for 1 mole weight of any ideal gas is equal to the universal gas constant.

Fuel-air ratios leaner than stoichiometric. - Consider the combustion of 1 mole weight of air of mean molecular weight of M_a. Then M_a is the mass of air and fM_a is the mass of fuel. The mass of oxygen consumed is

$$\frac{fM_a}{1+m}\left[\frac{16m}{2.016}+\frac{32}{12}\right]$$

and the masses of water vapor and carbon dioxide produced are

$\frac{18.016}{2.016} f \frac{M_a}{1+m} m$ and $\frac{44}{12} \frac{fM_a}{1+m}$, respectively.

The following equations connecting the thermodynamic properties of the mixture with those of the components are obtained by the use of the weighted averaging process:

$$\left.\begin{aligned} h_b &= \frac{1}{1+f}\left(h_a + f\,\frac{Am+B}{m+1}\right) \\ s_b &= \frac{1}{1+f}\left(s_a + f\,\frac{\alpha m+\beta}{m+1}\right) \end{aligned}\right\} \qquad (20)$$

$$\left.\begin{aligned} R_b &= \frac{R_a + \dfrac{fRm}{4.032(m+1)}}{1+f} \\ c_p &= \frac{c_{p_a} + f\,\dfrac{am+b}{m+1}}{1+f} \end{aligned}\right\} \qquad (21)$$

where

$$\left.\begin{aligned} A &= \frac{H_{H_2O} - \frac{1}{2} H_{O_2}}{2.016} ; \quad B = \frac{H_{CO_2} - H_{O_2}}{12} \\ \alpha &= \frac{S_{H_2O} - \frac{1}{2} S_{O_2}}{2.016} ; \quad \beta = \frac{S_{CO_2} - S_{O_2}}{12} \\ a &= \frac{c_{p_{H_2O}} - \frac{1}{2} c_{p_{O_2}}}{2.016} ; \quad b = \frac{c_{p_{CO_2}} - c_{p_{O_2}}}{12} \end{aligned}\right\} \qquad (22)$$

The values of h_a, s_a, c_{p_a}, A, B, α, β, a, and b are given in table II.

Richer than stoichiometric mixture. - The composition of the exhaust gas in the rich range is computed from the equilibrium equation

$$K = \frac{(CO)(H_2O)}{(H_2)(CO_2)} \qquad (11)$$

where K = 3.8. The units of concentration for the quantities in parentheses are taken as pound moles per pound mole of original combustion air.

One method of solving this equation for the components of exhaust gas is as follows: Let $(O_2)_a$ be the molal concentration of oxygen per pound mole of air and $(H_2O)_a$ be the molal concentration of water vapor in the air before combustion. If $(CO)'$, $(CO_2)'$, and $(H_2O)'$ represent the concentration of the exhaust gas per pound mole of combustion air on the assumption that the hydrogen is completely burned to H_2O, then the true composition of the exhaust gas in terms of these fictitious values is given by

$$\left.\begin{aligned} (CO) &= (CO)' - (H_2) \\ (H_2O) &= (H_2O)' - (H_2) \\ (CO_2) &= (CO_2)' + (H_2) \end{aligned}\right\} \qquad (23)$$

The quantities $(CO)'$, $(H_2O)'$, and $(CO_2)'$ can readily be calculated from the known oxygen, water vapor, and fuel quantities and are given by

$$\left.\begin{aligned} (CO_2)' &= 2(O_2)_a - \frac{M_a\ f}{12(m+1)} - \frac{M_a\ fm}{2.016(m+1)} \\ (CO)' &= \frac{M_a f}{6(m+1)} - 2(O_2)_a + \frac{M_a\ fm}{2.016(m+1)} \\ (H_2O)' &= \frac{M_a\ fm}{2.016(m+1)} + (H_2O)_a \end{aligned}\right\} \qquad (24)$$

Substitution of equations (23) into equation (11) and solution for (H_2) gives

$$(H_2) = \frac{\sqrt{\left[K(CO_2)'+(CO)'+(H_2O)'\right]^2+4(K-1)(CO)'(H_2O)'}-\left[K(CO_2)'+(CO)'+(H_2O)'\right]}{2(K-1)} \qquad (25)$$

The quantity (H_2) may now be determined from equation (25) and then (CO), (CO_2), and (H_2O) may be determined from equations (23).

The values of h_b, s_b, c_{p_b}, and R_b are obtained by taking the weighted average of the corresponding properties of the constituent gases as previously described, giving the relations

$$\left.\begin{aligned} h_b &= \frac{1}{1+f}\left\{h_a - \frac{(O_2)_a}{M_a} C + \frac{f}{(1+m)}(D + mE) + \frac{(H_2)}{M_a} F\right\} \\ s_b &= \frac{1}{1+f}\left\{s_a - \frac{(O_2)_a}{M_a} \Gamma + \frac{f}{(1+m)}(\delta + m\epsilon) + \frac{(H_2)}{M_a} \zeta\right\} \\ c_{p_b} &= \frac{1}{1+f}\left\{c_{p_a} - \frac{(O_2)_a}{M_a} c + \frac{f}{(1+m)}(d + me) + \frac{(H_2)}{M_a} \phi\right\} \\ R_b &= \frac{1}{1+f}\left\{R_a\left[1 - (O_2)_a\right] + R\frac{f}{(1+m)}\left(\frac{2.016 + 12m}{24.192}\right)\right\} \end{aligned}\right\} \qquad (26)$$

where

$$
\left.
\begin{aligned}
C &= H_{O_2} + 2H_{CO} - 2H_{CO_2} \\
D &= \left(2H_{CO} - H_{CO_2}\right)/12 \\
E &= \left(H_{H_2O} + H_{CO} - H_{CO_2}\right)/2.016 \\
F &= H_{CO_2} + H_{H_2} - H_{CO} - H_{H_2O} \\
\Gamma &= S_{O_2} + 2S_{CO} - 2S_{CO_2} \\
\delta &= \left(2S_{CO} - S_{CO_2}\right)/12 \\
\epsilon &= \left(S_{H_2O} + S_{CO} - S_{CO_2}\right)/2.016 \\
\zeta &= S_{CO_2} + S_{H_2} - S_{CO} - S_{H_2O} \\
c &= C_{p_{O_2}} + 2C_{p_{CO}} - 2C_{p_{CO_2}} \\
d &= \left(2C_{p_{CO}} - C_{p_{CO_2}}\right)/12 \\
e &= \left(C_{p_{H_2O}} + C_{p_{CO}} - C_{p_{CO_2}}\right)/2.016 \\
\phi &= C_{p_{CO_2}} + C_{p_{H_2}} - C_{p_{CO}} - C_{p_{H_2O}}
\end{aligned}
\right\} \qquad (27)
$$

The values of h_a, s_a, c_{p_a}, C, D, E, F, Γ, δ, ϵ, ζ, c, d, e, and ϕ are listed in table II. The values of the specific heat of water vapor used in the construction of figures 9 to 12 were those given in reference 5.

The values given in tables I and II include the contribution to the specific heat of water due to molecular stretching as described in reference 6. No change in figures 9 to 12 is necessary because the effect of this added term on the specific heat of exhaust gases is very small and affects γ only in the fourth place.

APPENDIX C

CONDITIONS FOR WHICH γ_t/γ_1 AND γ_h/γ_1 ARE FUNCTIONS ONLY OF PRESSURE RATIO

The purpose of this discussion is to show the conditions under which the ratio of the effective values of γ to the initial value γ_1 are functions of the pressure ratio p_1/p_2. Expressions for the effective values of γ will be derived.

The quantity called the entropy at 1 atmosphere is related to the temperature by

$$ds = c_p \frac{dT}{T}$$

For an isentropic process

$$\frac{Jc_p dT}{R_b T} = \frac{Jds}{R_b} = \frac{\gamma}{\gamma-1}\frac{dT}{T} = \frac{dp}{p} \tag{28}$$

$$\int_1^2 \frac{dT}{T} = \int_1^2 \frac{\gamma-1}{\gamma}\frac{dp}{p}$$

$$\log \frac{T_2}{T_1} = \log \frac{p_2}{p_1} - \int_1^2 \frac{1}{\gamma}\frac{dp}{p} \tag{29}$$

where these and subsequent logarithms are to the natural logarithmic base e. It is proposed to find first the conditions required for γ_t/γ_1 to be a function only of p_1/p_2.

From equation (7)

$$\log \frac{T_2}{T_1} = \left(1 - \frac{1}{\gamma_t}\right) \log \frac{p_2}{p_1}$$

When this expression for $\log\ T_2/T_1$ is equated to equation (29) and solution made for γ_t, there is obtained

$$\frac{\gamma_1}{\gamma_t} \log\left(\frac{p_1}{p_2}\right) = - \int_1^2 \frac{\gamma_1}{\gamma}\frac{dp}{p} \tag{30}$$

This relation shows that γ_t/γ_1 is a function of p_1/p_2 only when γ/γ_1 is a function of p/p_1. Thus

$$\gamma/\gamma_1 = f(p/p_1)$$

or

$$p/p_1 = F(\gamma/\gamma_1) \tag{31}$$

where f and F indicate function as yet not known. But

$$\frac{dp}{p_1} = \frac{dF(\gamma/\gamma_1)}{d\ (\gamma/\gamma_1)} \frac{d\gamma}{\gamma_1}$$

Therefore, equation (28) becomes

$$\frac{Jds}{R_b} = \frac{\gamma}{\gamma-1} \frac{dT}{T} = \frac{dp}{p} = \frac{\gamma/\gamma_1}{F(\gamma/\gamma_1)} \frac{dF(\gamma/\gamma_1)}{d(\gamma/\gamma_1)} \frac{d\gamma}{\gamma} \tag{32}$$

Since γ is a function only of T and is independent of any arbitrary starting point such as γ_1, the factor involving γ_1 must be equal to a constant. Therefore, a further condition that γ_t/γ_1 is a function only of p_1/p_2 is that

$$\frac{\gamma/\gamma_1}{F(\gamma/\gamma_1)} \frac{dF(\gamma/\gamma_1)}{d\ (\gamma/\gamma_1)} = \frac{1}{r} \tag{33}$$

where r is a constant. When this equation is integrated, there results

$$F(\gamma/\gamma_1) = \left(\frac{\gamma}{\gamma_1}\right)^{\frac{1}{r}}$$

From equation (31)

$$p/p_1 = \left(\frac{\gamma}{\gamma_1}\right)^{\frac{1}{r}}$$

or

$$\gamma/\gamma_1 = (p/p_1)^r \tag{34}$$

is the condition that γ_t/γ_1 is a function only of p_1/p_2. This condition may be restated in a more convenient form. Equation (32) becomes

$$\frac{J ds}{R_b} = \frac{d\gamma}{r\gamma}$$

On integration

$$r \frac{J(s-s_1)}{R_b} = \log \frac{\gamma}{\gamma_1} \tag{35}$$

Equation (35) is equivalent to equation (34) and indicates that γ_t/γ_1 is a function only of p_1/p_2 in the range where a plot of $\log \gamma$ against Js/R_b yields a straight line. The slope of this line gives the constant r.

Figure 13 shows $\log \gamma$ plotted against Js/R_b for the products of combustion of the following mixtures:

Air
Air + 0.0662 octane
Air + 0.10 octane
Air + 0.10 benzene

The gas temperatures are also shown in this figure.

It is noted that in each case the curves are substantially straight in the range of temperatures from 900° to 2500° F absolute and the slopes are substantially equal. An average value of r for the four curves shown is

$$r = -0.014$$

Substitution of $(p/p_1)^r$ for γ/γ_1 in equation (30) and integration yields

$$\frac{\gamma_t}{\gamma_1} = \frac{r \log \frac{p_1}{p_2}}{\left(\frac{p_1}{p_2}\right)^r - 1} \tag{36}$$

Values computed from this relation show excellent agreement with values given in figure 2(b).

It will now be shown over the same range of conditions (that is, where γ_t/γ_1 is a function only of p_1/p_2) that $\Delta h/RT_1$ is a function only of γ_1 and p_1/p_2. By definition

$$\Delta h = \int_1^2 c_p dT = \frac{R_b}{J}\int_1^2 \frac{\gamma}{\gamma-1} dT$$

For an isentropic process

$$\Delta h = \frac{R_b}{J}\int_1^2 T \frac{dp}{p}$$

But

$$T = T_1 \left(\frac{p}{p_1}\right)^{\frac{\gamma_t-1}{\gamma_t}}$$

where

$$\frac{\gamma_t}{\gamma_1} = \frac{r \log\left(\frac{p_1}{p}\right)}{\left(\frac{p_1}{p}\right)^r - 1} \tag{38}$$

Thus

$$\frac{J\Delta h}{R_b T_1} = \int_1^2 \left(\frac{p}{p_1}\right)^{-\frac{1}{\gamma_t}} \frac{dp}{p_1} \tag{39}$$

From equation (38)

$$\left(\frac{p}{p_1}\right)^{-\frac{1}{\gamma_t}} = e^{\frac{1}{r\gamma_1}\left[\left(\frac{p_1}{p}\right)^r - 1\right]}$$

On substitution in the equation for Δh

$$\frac{J\Delta h}{R_b T_1} = e^{-\frac{1}{r\gamma_1}} \int_1^2 e^{\frac{1}{r\gamma_1}\left(\frac{p_1}{p}\right)^r} \frac{dp}{p_1}$$

Thus $\Delta h/R_b T_1$ is seen to be a function of only γ_1 and p_1/p_2 in the range in which $\log \gamma$ is a straight line when plotted against Js/R_b.

An expression for γ_h/γ_1 will now be derived. From equation (5)

$$\frac{J\Delta h}{R_b T_1} = \int_1^2 \left(\frac{p}{p_1}\right)^{-\frac{1}{\gamma_h}} \frac{dp}{p_1}$$

where γ_h is constant during the integration. When this equation is subtracted from equation (39), there results

$$\int_1^2 \left[\left(\frac{p}{p_1}\right)^{-\frac{1}{\gamma_t}} - \left(\frac{p}{p_1}\right)^{-\frac{1}{\gamma_h}}\right] \frac{dp}{p_1} = 0 \qquad (40)$$

This equation will be solved to obtain an expression for γ_h as follows: Equation (40) may be written

$$\int \left(\frac{p}{p_1}\right)^{-\frac{1}{\gamma_1}} \left[\left(\frac{p}{p_1}\right)^{\frac{1}{\gamma_1} - \frac{1}{\gamma_t}} - \left(\frac{p}{p_1}\right)^{\frac{1}{\gamma_1} - \frac{1}{\gamma_h}}\right] \frac{dp}{p_1} = 0 \qquad (41)$$

A useful series for this analysis is

$$X^n = 1 + n \log X + \frac{n^2}{2!} \log^2 X + \frac{n^3}{3!} \log^3 X + \cdots \qquad (42)$$

Since $\frac{1}{\gamma_1} - \frac{1}{\gamma_t}$ and also $\frac{1}{\gamma_1} - \frac{1}{\gamma_h}$ are very small, the terms in which they are involved can be approximated by the first two terms of the series expansion. Thus

$$\int_1^2 \left(\frac{p}{p_1}\right)^{-\frac{1}{\gamma_1}} \left[-\frac{1}{\gamma_t} \log \frac{p}{p_1} + \frac{1}{\gamma_h} \log \frac{p}{p_1} \right] \frac{dp}{p_1} = 0$$

The term $\frac{1}{\gamma_t} \log \frac{p}{p_1}$ can be replaced by an expression obtained from equation (38)

$$\int_1^2 \left(\frac{p}{p_1}\right)^{-\frac{1}{\gamma_1}} \left\{ -\frac{1}{\gamma_1 r} \left[1 - \left(\frac{p_1}{p}\right)^r \right] + \frac{1}{\gamma_h} \log \frac{p}{p_1} \right\} \frac{dp}{p_1} = 0$$

As r is very small $(p_1/p)^r$ can be replaced by the first three terms of its series expansion. (See equation (42).) Thus

$$\int_1^2 \left(\frac{p}{p_1}\right)^{-\frac{1}{\gamma_1}} \left[\frac{1}{\gamma_1} \log \frac{p_1}{p} + \frac{r}{2\gamma_1} \log^2 \frac{p_1}{p} + \frac{1}{\gamma_h} \log \frac{p}{p_1} \right] \frac{dp}{p_1} = 0$$

When solution is made for γ_1/γ_h there results

$$\frac{\gamma_1}{\gamma_h} = 1 + \frac{r}{2} \frac{\int_1^2 \left(\frac{p}{p_1}\right)^{-\frac{1}{\gamma_1}} \log^2 \frac{p_1}{p} \frac{dp}{p_1}}{\int_1^2 \left(\frac{p}{p_1}\right)^{-\frac{1}{\gamma_1}} \log \frac{p_1}{p} \frac{dp}{p_1}}$$

The integrations indicated in the equation for γ_1/γ_h can be explicitly carried out. The following approximate evaluation of these integrals is more expedient in the present circumstances.

$$\int_1^2 \left(\frac{p}{p_1}\right)^{-\frac{1}{\gamma_1}} \log^2 \frac{p_1}{p} \frac{dp}{p_1} = \int_1^2 \left(\frac{p}{p_1}\right)^{1-\frac{1}{\gamma_1}} \log^2 \frac{p_1}{p} \, d \log \frac{p}{p_1}$$

For the purpose at hand it is sufficiently accurate to replace the term $\left(\frac{p}{p_1}\right)^{1-\frac{1}{\gamma_1}}$ by the first three terms of the series expansion (equation (42)). Thus

$$\int_1^2 \left(\frac{p}{p_1}\right)^{-\frac{1}{\gamma_1}} \log^2 \frac{p_1}{p} \frac{dp}{p_1}$$

$$= \int_1^2 \left[\log^2 \frac{p}{p_1} + \left(1 - \frac{1}{\gamma_1}\right)\log^3 \frac{p}{p_1} + \frac{1}{2}\left(1 - \frac{1}{\gamma_1}\right)^2 \log^4 \frac{p}{p_1}\right] d \log \frac{p}{p_1}$$

$$= \frac{1}{3} \log^3 \frac{p_2}{p_1} + \frac{1}{4}\left(1 - \frac{1}{\gamma_1}\right)\log^4 \frac{p_2}{p_1} + \frac{1}{10}\left(1 - \frac{1}{\gamma_1}\right)^2 \log^5 \frac{p_2}{p_1}$$

Similarly

$$-\int_1^2 \left(\frac{p}{p_1}\right)^{-\frac{1}{\gamma_1}} \log \frac{p_1}{p} \frac{dp}{p_1} = \frac{1}{2} \log^2 \frac{p_2}{p_1} + \frac{1}{3}\left(1 - \frac{1}{\gamma_1}\right)\log^3 \frac{p_2}{p_1} + \frac{1}{8}\left(1 - \frac{1}{\gamma}\right)^2 \log^4 \frac{p_2}{p_1}$$

Thus

$$\frac{\gamma_1}{\gamma_h} = 1 - \frac{r}{3} \log \frac{p_2}{p_1} \left[\frac{1 + \frac{3}{4}\left(1 - \frac{1}{\gamma_1}\right) \log \frac{p_2}{p_1} + \frac{3}{10}\left(1 - \frac{1}{\gamma_1}\right)^2 \log^2 \frac{p_2}{p_1}}{1 + \frac{2}{3}\left(1 - \frac{1}{\gamma_1}\right) \log \frac{p_2}{p_1} + \frac{1}{4}\left(1 - \frac{1}{\gamma_1}\right)^2 \log^2 \frac{p_2}{p_1}}\right]$$

When $p_1/p_2 = 1$, the bracketed quantity reduces to 1. When $p_1/p_2 = 10$ and $\gamma_1 = 1.33$, the bracketed quantity is equal to 0.951. Thus it is of sufficient accuracy to take the bracketed quantity equal to unity. Then

$$\frac{\gamma_1}{\gamma_h} = 1 + \frac{r}{3} \log \frac{p_1}{p_2} \qquad (43)$$

Since the last term is of the order of 0.01 or less over the usual range of pressure ratios, the further approximation

$$\frac{\gamma_h}{\gamma_1} = 1 - \frac{r}{3} \log \frac{p_1}{p_2}$$

is permissible. Values computed from this relation agree closely with values obtained from figure 2(a).

Another form in which γ_h/γ_1 may be written is

$$\frac{\gamma_h}{\gamma_1} = \left(\frac{p_1}{p_2}\right)^{-\frac{r}{3}}$$

This relation is seen to reduce to the previous form when the first two terms of the series expansion are taken.

Other forms for the γ ratios may be found that may be useful. The quantity γ_t/γ_1 reduces to the following expression when the first three terms of the series expansion for $(p_1/p_2)^2$ are used.

$$\frac{\gamma_1}{\gamma_t} = 1 + \frac{r}{2} \log \frac{p_1}{p_2}$$

This relation may also be represented to a sufficient degree of accuracy by the following equation. (See equation (42).)

$$\frac{\gamma_t}{\gamma_1} = \left(\frac{p_1}{p_2}\right)^{-\frac{r}{2}}$$

To summarize: In the region where $\log \gamma$ plots as a straight line against Js/R_b and when r the slope of this line is small compared with unity

$$\frac{\gamma_2}{\gamma_1} = \left(\frac{p_1}{p_2}\right)^{-r} \tag{44}$$

$$\frac{\gamma_t}{\gamma_1} = \left(\frac{p_1}{p_2}\right)^{-\frac{r}{2}} \tag{45}$$

$$\frac{\gamma_h}{\gamma_1} = \left(\frac{p_1}{p_2}\right)^{-\frac{r}{3}} \tag{46}$$

For any given value of Js/R_b the difference between the values of $\log \gamma$ given by the curve and the straight line represents the percentage error in γ when the straight line is used as an approximation for the curve (fig. 13). This result follows from the relation $d \log \gamma = d\gamma/\gamma$. It is noted that in the range from 900° F absolute (440° F) to 2500° F absolute (2140° F) the error in the value of γ given by the straight line is less than 0.1 percent.

APPENDIX D

THE CONSTRUCTION OF THE CORRECTION CHARTS

Three terms are required to correct thermodynamic quantities for changes in gas constant and ratio of specific heats. The correction factor K_R is equal to the ratio of the gas constant for exhaust gas to that for air.

$$K_R = \frac{R_b}{53.35}$$

The correction factor K_γ for the effect of changes in γ on W_{th} is given by

$$K_\gamma = 1 + \left(\frac{\gamma}{W_{th}} \frac{\partial W_{th}}{\partial \gamma}\right) \frac{\Delta \gamma}{\gamma_s}$$

γ_s being taken as 1.329, the value of γ for air at 1980° F absolute. The corrections shown in figures 9 to 12 are thus actually set up for 1980° F absolute. The error involved in the use of this correction factor for other initial temperatures is negligible.

The correction factor for mass flow K_μ is given by

$$K_\mu = 1 + \left(\frac{\gamma}{M} \frac{\partial M}{\partial \gamma}\right) \frac{\Delta \gamma}{\gamma_s}$$

The values of the two logarithmic partial derivatives $\frac{\gamma}{W_{th}} \frac{\partial W_{th}}{\partial \gamma}$ and $\frac{\gamma}{M} \frac{\partial M}{\partial \gamma}$, evaluated for a γ of 1.33 using the formulas for constant specific heats, are shown in figure 14. The values of the correction factors are practically independent of the value of γ used in these computations.

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TABLE I

THERMODYNAMIC FUNCTIONS OF EXHAUST-GAS CONSTITUENTS

IN THE STANDARD STATE

T (°K)	T (°F abs.)	H_2 (a)	CO (a)	N_2	O_2	H_2O (b)	CO_2 (b)	A (c)
		Enthalpy, $H - E_o$, (Btu)/(lb mole)						
300	540	3,665.3	3,753.4	3,752.6	3,748.5	4,284.8	4,061.0	2,682.3
400	720	4,915.6	5,010.1	5,007.6	5,026.3	5,743.8	5,755.5	3,576.4
500	900	6,172.7	6,281.6	6,272.6	6,344.3	7,237.9	7,608.2	4,470.5
600	1080	7,431.1	7,577.3	7,556.2	7,704.0	8,776	9,590.2	5,364.6
700	1260	8,696.3	8,903.2	8,865.9	9,105.5	10,368	11,678	6,258.7
800	1440	9,967.0	10,261.1	10,204.6	10,542.2	12,009	13,855	7,152.8
900	1620	11,246.4	11,649.4	11,572.4	12,009	13,706	16,107	8,046.9
1000	1800	12,539	13,065	12,967	13,499	15,460	18,419	8,941.1
1100	1980	13,847	14,505	14,387	15,009	17,266	20,786	9,835.1
1200	2160	15,171	15,967	15,829	16,538	19,140	23,195	10,729
1300	2340	16,513	17,447	17,291	18,080	21,046	25,643	11,623
1400	2520	17,874	18,943	18,769	19,636	23,013	28,123	12,517
1500	2700	19,254	20,453	20,263	21,205	25,017	30,631	13,412
Reference--	---------	1	2	2	3, 4	5, 6	7	
		Entropy at 1 atmosphere pressure, S, (Btu)/(lb mole)(°F)						
300	540	31.269	47.357	45.828	49.061	45.179	51.140	28.332
400	720	33.267	49.366	47.833	51.121	47.509	53.842	29.761
500	900	34.826	50.942	49.401	52.740	49.361	56.135	30.870
600	1080	36.101	52.254	50.701	54.117	50.919	58.141	31.775
700	1260	37.184	53.389	51.822	55.314	52.280	59.929	32.541
800	1440	[d]38.126	54.396	52.815	56.381	53.499	61.543	33.204
900	1620	38.964	55.304	53.710	57.342	54.608	63.016	33.789
1000	1800	39.721	56.133	54.527	58.214	55.634	64.370	34.313
1100	1980	40.413	56.896	55.279	59.013	56.590	65.623	34.786
1200	2160	41.053	57.602	55.976	59.751	57.490	66.787	35.218
1300	2340	41.650	[d]58.261	56.626	60.437	58.343	67.875	35.616
1400	2520	42.210	58.876	57.234	61.075	59.151	68.897	35.984
1500	2700	42.739	59.455	57.807	61.680	59.921	69.858	36.327
Reference--	---------	1	2	2	3, 4	5, 6	7	
		Specific heat at constant pressure, C_p, (Btu)/(lb mole)(°F)						
300	540	6.896	6.964	6.960	7.021	8.030	8.908	4.967
400	720	6.974	7.013	6.991	7.197	8.192	9.885	4.967
500	900	6.992	7.122	7.071	7.434	8.425	10.676	4.967
600	1080	7.008	7.279	7.200	7.675	8.690	11.324	4.967
700	1260	7.035	7.455	7.355	7.890	8.974	11.862	4.967
800	1440	7.079	7.629	7.516	8.069	9.273	12.312	4.967
900	1620	7.141	7.792	7.676	8.216	9.580	12.689	4.967
1000	1800	7.220	7.936	7.821	8.341	9.891	13.005	4.967
1100	1980	7.314	8.061	7.952	8.445	10.196	13.27	4.967
1200	2160	7.408	8.175	8.069	8.534	10.492	13.50	4.967
1300	2340	7.508	8.269	8.169	8.612	10.776	13.69	4.967
1400	2520	7.613	8.346	8.252	8.677	11.043	13.86	4.967
1500	2700	7.718	8.422	8.334	8.742	11.291	14.00	4.967
Reference--	---------	1	2	2	3, 4	5, 6	7	Calc.

[a]Taking E_o = 0 for CO_2, H_2O, O_2, E_o for CO and H_2 has been assumed to have the values:

gas	E_o
CO	119,626
H_2	102,243

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

[b]Values not appearing in the original references, calculated by means of the identity $H = TS + F$.

[c]Calculated by ideal gas law, $C_p = 4.967$, $S = \frac{5}{2} R \log T$.

[d]Original reference in error. Tabulated value interpolated and relatively accurate to ±0.001 (Btu)/(lb mole)(°F).

TABLE II

DERIVED THERMODYNAMIC FUNCTIONS OF GASES

T (°K)	T (°F abs.)	Factors for calculating enthalpy h, (Btu)/(lb)						
		A	B	C (1)	D (1)	E (1)	F (1)	h_a
300	540	1195.8	26.042	242,385.3	20,224.8	61,311.2	-17,694.9	129.13
400	720	1602.6	60.767	242,787.5	20,293.1	61,817.8	-17,465.8	172.48
500	900	2016.9	105.325	242,943.1	20,350.6	62,270.7	-17,121.6	216.40
600	1080	2442.8	157.18	242,930.2	20,401.4	62,693.3	-16,712.2	261.13
700	1260	2884.9	214.38	242,808	20,448.4	63,105.1	-16,279.6	306.85
800	1440	3343.3	276.07	242,606	20,493	63,514.0	-15,831.5	353.62
900	1620	3821.0	341.50	242,346	20,537	63,927.0	-15,385.0	401.38
1000	1800	4321.6	410.00	242,043	20,580	64,352.0	-14,950.0	450.04
1100	1980	4842.9	481.42	241,699	20,623	64,788.0	-14,521.0	499.52
1200	2160	5388.6	554.75	241,334	20,666	65,244.0	-14,124.0	549.73
1300	2340	5956.6	630.25	240,940	20,709	65,713.0	-13,720.0	600.58
1400	2520	6545.1	707.25	240,528	20,751	66,202.0	-13,342.0	651.96
1500	2700	7152.6	785.50	240,101	20,794	66,701.0	-12,968.0	703.86
		Factors for calculating entropy s, (Btu)/(lb)(°F)						
		α	β	Γ	δ	ε	ζ	s_a
300	540	10.242	0.1733	41.495	3.6312	20.534	-10.121	1.5992
400	720	10.887	.2268	42.169	3.7408	21.345	-9.766	1.6686
500	900	11.404	.2829	42.354	3.8124	21.909	-9.315	1.7229
600	1080	11.835	.3353	42.343	3.8639	22.337	-8.931	1.7682
700	1260	12.214	.3846	42.234	3.9041	22.689	-8.556	1.8073
800	1440	12.554	.4302	42.087	3.9374	22.992	-8.226	1.8420
900	1620	12.865	.4728	41.918	3.9660	23.262	-7.932	1.8733
1000	1800	13.159	.5130	41.740	3.9913	23.511	-7.676	1.9018
1100	1980	13.434	.5508	41.559	4.0141	23.741	-7.450	1.9280
1200	2160	13.698	.5863	41.381	4.0348	23.961	-7.252	1.9523
1300	2340	13.950	.6198	41.209	4.0539	24.171	-7.079	1.9749
1400	2520	14.193	.6518	41.033	4.0713	24.370	-6.920	1.9960
1500	2700	14.426	.6815	40.874	4.0877	24.563	-6.779	2.0159
		Factors for calculating specific heat at constant pressure c_p, (Btu)/(lb)(°F)						
		a	b	c	d	e	ø	c_{p_a}
300	540	2.242	0.1573	3.133	0.4183	3.019	0.810	0.2400
400	720	2.278	.2240	1.453	.3451	2.639	1.654	.2421
500	900	2.336	.2702	.326	.2973	2.416	2.121	.2460
600	1080	2.407	.3041	-.415	.2695	2.304	2.363	.2512
700	1260	2.495	.3310	-.924	.2540	2.266	2.468	.2569
800	1440	2.599	.3536	-1.297	.2455	2.277	2.489	.2626
900	1620	2.714	.3728	-1.578	.2412	2.323	2.458	.2679
1000	1800	2.838	.3887	-1.797	.2389	2.392	2.398	.2727
1100	1980	2.963	.4021	-1.973	.2377	2.429	2.327	.2770
1200	2160	3.088	.4138	-2.116	.2375	2.563	2.241	.2808
1300	2340	3.209	.4232	-2.230	.2373	2.656	2.153	.2841
1400	2520	3.326	.4319	-2.351	.2360	2.743	2.084	.2868
1500	2700	3.433	.4382	-2.414	.2370	2.834	2.005	.2895

[1]The value for E_0 of H_2 and CO have been added.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TABLE III - THEORETICAL WORK AVAILABLE IN AN ISENTROPIC EXPANSION

[Data from this table were used in preparing figure 3 of report.]

Pressure ratio, p_1/p_2	Ratio of specific heats at initial temperature, γ_1						
	1.28	1.30	1.32	1.34	1.36	1.38	1.40
	Available work, W_{th}/RbT_1						
1.2	0.1787	0.1785	0.1783	0.1782	0.1780	0.1778	0.1776
1.4	.3243	.3237	.3231	.3225	.3219	.3213	.3208
1.6	.4465	.4453	.4441	.4429	.4418	.4407	.4397
1.8	.5512	.5493	.5475	.5457	.5440	.5424	.5408
2.0	.6426	.6400	.6375	.6351	.6328	.6305	.6284
2.5	.8290	.8246	.8204	.8164	.8125	.8087	.8050
3	.9744	.9683	.9624	.9567	.9513	.9460	.9410
3.5	1.0926	1.0849	1.0774	1.0703	1.0634	1.0567	1.0504
4	1.1917	1.1824	1.1735	1.1649	1.1567	1.1487	1.1411
5	1.3503	1.3382	1.3266	1.3155	1.3048	1.2946	1.2847
6	1.4739	1.4593	1.4453	1.4320	1.4192	1.4070	1.3952
7	1.5743	1.5575	1.5415	1.5261	1.5114	1.4974	1.4839
8	1.6583	1.6395	1.6216	1.6045	1.5881	1.5724	1.5574
9	1.7332	1.7096	1.6899	1.6712	1.6533	1.6362	1.6198
10	1.7928	1.7706	1.7494	1.7292	1.7099	1.6915	1.6738

TABLE IV- RATIO OF SPECIFIC HEATS OF COMBUSTION GASES

[Data from this table were used in preparing figure 4 of report.]

Temperature (° F absolute)	Fuel-air ratio								
	0	0.03	0.04	0.05	0.06	0.07	0.08	0.10	0.12
	Ratio of specific heats, γ								
Hydrogen-carbon ratio, 0.084									
1080	1.3755	1.3568	1.3511	1.3456	1.3403	1.3353	1.3357	1.3478	1.3580
1260	1.3640	1.3551	1.3392	1.3337	1.3284	1.3209	1.3237	1.3361	1.3466
1440	1.3534	1.3342	1.3284	1.3228	1.3175	1.3124	1.3128	1.3252	1.3359
1620	1.3439	1.3247	1.3207	1.3133	1.3080	1.3029	1.3033	1.3156	1.3262
1800	1.3357	1.3166	1.3108	1.3052	1.2998	1.2947	1.2951	1.3072	1.3178
1980	1.3290	1.3096	1.3038	1.2982	1.2929	1.2879	1.2883	1.3003	1.3108
2160	1.3230	1.3037	1.2979	1.2923	1.2870	1.2819	1.2821	1.2938	1.3041
2340	1.3205	1.2987	1.2929	1.2873	1.2820	1.2770	1.2771	1.2886	1.2988
Hydrogen-carbon ratio, 0.189									
1080	1.3755	1.3559	1.3501	1.3445	1.3393	-------	1.3451	1.3587	1.3635
1260	1.3640	1.3442	1.3383	1.3327	1.3274	-------	1.3324	1.3439	1.3528
1440	1.3534	1.3333	1.3273	1.3217	1.3164	-------	1.3225	1.3331	1.3422
1620	1.3439	1.3237	1.3177	1.3120	1.3067	-------	1.3127	1.3233	1.3326
1800	1.3357	1.3154	1.3093	1.3037	1.2983	-------	1.3041	1.3148	1.3240
1980	1.3290	1.3082	1.3022	1.2964	1.2910	-------	1.2972	1.3078	1.3172
2160	1.3230	1.3021	1.2960	1.2902	1.2848	-------	1.2902	1.3006	1.3098
2340	1.3205	1.2969	1.2907	1.2849	1.2794	-------	1.2850	1.2953	1.3045

National Advisory Committee
for Aeronautics

TABLE V - GAS CONSTANT OF COMBUSTION GASES

[Data from this table were used in preparing figure 5 of report.]

Fuel-air ratio	Hydrogen-carbon ratio						
	0.084	0.100	0.125	0.150	0.175	0.189	0.200
	Gas constant R_b, (ft lb)/(lb)(°F)						
0.01	53.12	53.17	53.245	53.32	53.39	53.43	53.46
.02	52.89	52.99	53.14	53.29	53.43	53.50	53.56
.03	52.66	52.81	53.04	53.25	53.46	53.57	53.66
.04	52.45	52.64	52.94	53.22	53.50	53.64	53.77
.05	52.23	52.47	52.84	53.19	53.53	53.71	53.86
.06	52.01	52.30	52.74	53.16	53.57	53.78	53.95
.07	51.805	52.14	52.65	53.26	54.03	54.45	54.77
.08	53.23	52.86	53.82	54.73	55.60	56.08	56.44
.09	53.39	54.09	55.155	56.17	57.15	57.68	58.08
.10	54.52	55.30	56.47	57.59	58.67	59.24	59.69
.11	55.63	56.48	57.76	58.98	60.15	60.79	61.27
.12	56.73	57.65	59.03	60.35	61.61	62.30	62.82

TABLE VI - TEMPERATURE CHANGE IN AN ISENTROPIC EXPANSION

[Data from this table were used in preparing figure 6 of report.]

Pressure ratio p_1/p_2	Ratio of specific heats at initial temperature, γ_1						
	1.28	1.30	1.32	1.34	1.36	1.38	1.40
	Ratio of final temperature to initial temperature, T_2/T_1						
1.2	0.9607	0.9586	0.9566	0.9547	0.9527	0.9509	0.9491
1.4	.9285	.9248	.9212	.9177	.9143	.9111	.9079
1.6	.9013	.8962	.8914	.8867	.8821	.8777	.8735
1.8	.8778	.8717	.8657	.8600	.8545	.8492	.8441
2.0	.8572	.8502	.8433	.8368	.8305	.8244	.8185
2.5	.8148	.8059	.7974	.7892	.7813	.7738	.7666
3.0	.7814	.7712	.7615	.7521	.7432	.7346	.7263
3.5	.7541	.7488	.7322	.7220	.7122	.7029	.6939
4.0	.7308	.7189	.7075	.6966	.6862	.6762	.6667
5	.6935	.6804	.6679	.6560	.6446	.6338	.6234
6	.6641	.6501	.6368	.6242	.6122	.6008	.5899
7	.6401	.6255	.6116	.5985	.5860	.5741	.5628
8	.6198	.6047	.5904	.5769	.5641	.5519	.5403
9	.6024	.5869	.5723	.5585	.5454	.5330	.5211
10	.5873	.5715	.5566	.5426	.5292	.5166	.5046

TABLE VII - ENTHALPY CHANGE AS A FUNCTION OF TEMPERATURE

[Data from this table were used in preparing figure 7 of report.]

Temperature ratio, T_2/T_1	Ratio of specific heats at initial temperature, γ_1						
	1.28	1.30	1.32	1.34	1.36	1.38	1.40
	Change in enthalpy, $-J\Delta h/R_bT_1$						
0.99	0.04566	0.04329	0.04121	0.03938	0.03775	0.03629	0.03497
.98	.09123	.08650	.08236	.07870	.07545	.07254	.06992
.97	.1367	.1296	.1234	.1180	.1131	.1087	.1048
.96	.1821	.1727	.1644	.1572	.1507	.1449	.1397
.95	.2274	.2157	.2054	.1963	.1882	.1810	.1745
.90	.4523	.4293	.4091	.3912	.3752	.3609	.3480
.85	.6745	.6406	.6108	.5843	.5607	.5394	.5204
.80	.8934	.8491	.8101	.7754	.7446	.7167	.6915
.75	1.1100	1.0550	1.0075	.9648	.9266	.8922	.8611
.70	1.3228	1.2590	1.2025	1.1522	1.1072	1.0666	1.0297
.65	1.5328	1.4597	1.3950	1.3374	1.2856	1.2389	1.1965
.60	1.7401	1.6580	1.5854	1.5206	1.4623	1.4096	1.3619
.55	--------	--------	--------	1.7021	1.6372	1.5788	1.5258

TABLE VIII - CRITICAL PRESSURE RATIO AND CRITICAL MASS-FLOW FACTOR

[Data from this table were used in preparing figure 8 of report.]

	Ratio of specific heats at initial temperature, γ_1						
	1.28	1.30	1.32	1.34	1.36	1.38	1.40
Critical pressure ratio, p_1/p_2	1.8277	1.8403	1.8525	1.8648	1.8768	1.8892	1.9015
$\frac{M_{cr}\sqrt{gR_bT_1}}{p_1A}$	0.66454	0.66809	0.67179	0.67533	0.67892	0.68232	0.68575

National Advisory Committee
for Aeronautics

TABLE IX - IDEAL WORK IN THE EXPANSION OF AIR

[Data from this table were used in preparing figure 9 of report.]

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

Pressure ratio, p_1/p_2	Initial temperature, T_1, °F absolute															
	1200	1300	1400	1500	1600	1700	1800	1900	2000	2100	2200	2300	2400	2500	2600	2700
	Ideal work, (Btu)/(lb)															
1.025	2.025	2.194	2.363	2.531	2.698	2.867	3.033	3.204	3.375	3.545	3.716	3.880	4.055	4.219	4.386	4.558
1.050	3.988	4.320	4.655	4.987	5.321	5.652	5.988	6.320	6.653	6.987	7.315	7.649	7.979	8.315	8.652	8.983
1.075	5.895	6.386	6.879	7.371	7.862	8.353	8.844	9.335	9.833	10.32	10.81	11.31	11.80	12.30	12.78	13.28
1.100	7.742	8.389	9.036	9.684	10.33	10.98	11.63	12.27	12.91	13.57	14.21	14.86	15.51	16.15	16.81	17.45
1.125	9.537	10.34	11.13	11.93	12.73	13.53	14.33	15.12	15.92	16.72	17.51	18.31	19.11	19.90	20.71	21.50
1.150	11.29	12.23	13.17	14.16	15.06	16.01	16.95	17.89	18.84	19.79	20.73	21.67	22.62	23.56	24.51	25.45
1.20	14.64	15.86	17.09	18.31	19.54	20.77	22.00	23.22	24.45	25.68	26.90	28.52	29.36	30.58	31.82	33.03
1.25	17.82	19.31	20.81	22.30	23.80	25.29	26.79	28.28	29.77	31.27	32.76	34.25	35.76	37.24	38.75	40.24
1.30	20.84	22.59	24.34	26.09	27.83	29.58	31.34	33.09	34.83	36.58	38.34	40.09	41.85	43.59	45.35	47.09
1.4	26.47	28.69	30.91	33.14	35.36	37.58	39.81	42.05	44.27	46.49	48.72	50.96	53.17	55.41	57.65	59.87
1.5	31.61	34.25	36.92	39.58	42.24	44.90	47.57	50.22	52.90	55.56	58.23	60.90	63.55	66.23	69.91	71.56
1.6	36.32	39.37	42.44	45.50	48.56	51.63	54.70	57.76	60.84	63.91	66.97	70.05	73.10	76.19	79.27	82.32
1.7	40.67	44.10	47.54	50.97	54.40	57.85	61.29	64.72	68.15	71.62	75.07	78.51	81.93	85.40	88.24	92.29
1.8	44.71	48.47	52.27	56.03	59.84	63.63	67.39	71.19	74.97	78.79	82.58	86.35	90.15	93.95	97.73	101.49
1.9	48.47	52.56	56.67	60.77	64.89	69.01	73.10	77.23	81.33	85.47	89.60	93.69	97.77	101.95	106.05	110.13
2.0	51.98	56.39	60.78	65.20	69.61	74.03	78.45	82.88	87.40	91.70	96.15	100.56	104.97	109.44	113.83	118.23
2.25	59.90	64.97	70.04	75.15	80.23	85.35	90.46	95.57	100.66	105.77	110.88	116.00	121.12	126.22	131.35	136.42
2.50	66.70	72.39	78.05	83.76	89.45	95.18	100.87	106.57	112.41	118.00	123.69	129.44	135.09	140.85	146.58	152.24
3.0	78.11	84.76	91.43	98.14	104.81	111.54	118.24	124.97	131.65	138.37	145.14	151.87	158.53	165.30	171.98	178.70
3.5	87.31	94.75	102.25	109.73	117.23	124.79	132.31	139.81	147.35	154.90	162.43	169.99	177.52	185.12	192.61	200.07
4.0	94.86	103.00	111.15	119.41	127.54	135.80	143.99	152.23	160.60	168.69	176.84	185.15	193.32	201.59	209.84	217.97
5.0	106.97	116.23	125.43	134.73	144.01	153.33	162.67	171.95	181.48	190.60	199.69	209.26	219.10	227.97	237.41	246.62
6.0	116.27	126.41	136.44	146.63	156.74	166.95	177.09	187.27	197.64	207.65	217.84	228.03	238.21	248.48	258.61	268.66
7.0	123.79	134.59	145.30	156.18	166.98	177.88	188.75	199.59	210.71	221.36	232.34	243.14	254.09	265.17	276.01	286.75
8.0	130.02	141.43	152.87	164.18	175.52	187.09	198.45	209.97	221.59	232.92	244.36	255.90	267.31	278.93	290.43	301.74
9.0	135.32	147.29	158.96	171.03	182.81	194.82	206.74	218.75	230.89	242.71	254.46	266.68	278.64	290.71	302.72	314.51
10.0	139.89	152.27	164.38	176.85	189.08	201.56	213.92	226.28	238.93	251.10	263.59	276.02	288.41	301.07	313.52	325.74

TABLE X - IDEAL MASS FLOW OF AIR

[Data from this table were used in preparing figure 11 of report.]

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

Pressure ratio, P_1/P_2	Initial temperature, T_1, °F absolute															
	1200	1300	1400	1500	1600	1700	1800	1900	2000	2100	2200	2300	2400	2500	2600	2700
	Ideal mass flow, M, (lb)/(sec)(sq in.)(in. Hg initial pressure)															
1.025	0.002400	0.002306	0.002222	0.002146	0.002077	0.002015	0.001957	0.001906	0.001858	0.001813	0.001772	0.001732	0.001697	0.001661	0.001629	0.001599
1.050	.003309	.003179	.003063	.002959	.002865	.002779	.002701	.002628	.002562	.002500	.002442	.002388	.002337	.002290	.002246	.002204
1.075	.003955	.003799	.003630	.003535	.003422	.003319	.003225	.003139	.003060	.002985	.002916	.002852	.002792	.002736	.002682	.002621
1.100	.004457	.004281	.004124	.003984	.003856	.003741	.003635	.003537	.003447	.003363	.003286	.003213	.003145	.003081	.003021	.002964
1.125	.004866	.004674	.004502	.004349	.004209	.004083	.003968	.003860	.003763	.003672	.003586	.003507	.003432	.003362	.003297	.003235
1.150	.005210	.005003	.004819	.004661	.004505	.004370	.004246	.004131	.004026	.003928	.003837	.003752	.003673	.003597	.003527	.003461
1.20	.005751	.005522	.005320	.005137	.004972	.004822	.004685	.004558	.004442	.004333	.004232	.004167	.004051	.003968	.003891	.003817
1.25	.006159	.005914	.005696	.005500	.005323	.005162	.005014	.004879	.004753	.004627	.004528	.004428	.004336	.004246	.004163	.004084
1.3	.006473	.006216	.005985	.005779	.005563	.005422	.005240	.005124	.004992	.004857	.004732	.004651	.004528	.004458	.004371	.004289
1.4	.006916	.006635	.006392	.006167	.005970	.005784	.005618	.005466	.005327	.005193	.005074	.004958	.004855	.004752	.004659	.004571
1.5	.007181	.006893	.006633	.006404	.006192	.006006	.005829	.005673	.005522	.005390	.005260	.005145	.005029	.004930	.004833	.004742
1.6	.007352	.007050	.006790	.006548	.006338	.006139	.005965	.005798	.005650	.005508	.005381	.005256	.005146	.005036	.004937	.004843
1.7	.007438	.007138	.006857	.006627	.006399	.006212	.006021	.005865	.005702	.005571	.005429	.005315	.005192	.005093	.004991	.004897
1.8	.007482	.007179	.006906	.006664	.006443	.006246	.006061	.005896	.005739	.005598	.005463	.005340	.005224	.005116	.005014	.004919
Critical pressure ratio →	1.881	1.878	1.874	1.871	1.868	1.864	1.862	1.860	1.857	1.855	1.853	1.852	1.850	1.849	1.847	1.847
Critical mass flow, M_{cr} →	0.007492	0.007187	0.006915	0.006670	0.006454	0.006250	0.006067	0.005899	0.005744	0.005601	0.005468	0.005343	0.005228	0.005118	0.005016	0.004921

TABLE XI - IDEAL POWER FOR AIR

[Data from this table were used in preparing figure 12 of report.]

Pressure ratio, P_1/P_2	Initial temperature, T_1, °F absolute															
	1200	1300	1400	1500	1600	1700	1800	1900	2000	2100	2200	2300	2400	2500	2600	2700
	Ideal power, (hp)/(sq in.)(in. Hg initial pressure)															
1.025	0.00687	0.00715	0.00742	0.00768	0.00793	0.00817	0.00840	0.00864	0.00887	0.00909	0.00931	0.00951	0.00973	0.00991	0.0101	0.0103
1.050	.0187	.0194	.0202	.0209	.0216	.0222	.0229	.0235	.0241	.0247	.0253	.0258	.0264	.0269	.0275	.0280
1.075	.0330	.0343	.0391	.0369	.0381	.0392	.0403	.0414	.0426	.0436	.0446	.0456	.0466	.0476	.0485	.0680
1.100	.0489	.0508	.0528	.0546	.0564	.0581	.0598	.0614	.0630	.0645	.0660	.0675	.0690	.0704	.0718	.0732
1.125	.0656	.0683	.0709	.0734	.0758	.0781	.0804	.0825	.0848	.0869	.0888	.0908	.0928	.0946	.0966	.0984
1.150	.0832	.0866	.0898	.0933	.0960	.0990	.1018	.1046	.1073	.1099	.1125	.1150	.1175	.1199	.1223	.1246
1.20	.1191	.1239	.1286	.1331	.1374	.1417	.1458	.1497	.1536	.1574	.1610	.1681	.1682	.1716	.1751	.1784
1.25	.1552	.1615	.1678	.1735	.1792	.1847	.1900	.1951	.2001	.2047	.2098	.2145	.2193	.2237	.2282	.2324
1.30	.1914	.1986	.2065	.2133	.2205	.2269	.2336	.2398	.2460	.2513	.2578	.2637	.2691	.2748	.2804	.2857
1.4	.2600	.2693	.2804	.2891	.2993	.3075	.3170	.3251	.3338	.3415	.3498	.3573	.3651	.3724	.3799	.3871
1.5	.3222	.3339	.3473	.3585	.3705	.3814	.3924	.4030	.4131	.4236	.4328	.4437	.4516	.4618	.4711	.4800
1.6	.3795	.3926	.4091	.4214	.4363	.4483	.4618	.4736	.4860	.4979	.5091	.5208	.5312	.5427	.5536	.5639
1.7	.4296	.4452	.4629	.4777	.4935	.5083	.5223	.5369	.5496	.5643	.5754	.5902	.6005	.6152	.6270	.6392
1.8	.4759	.4922	.5125	.5281	.5464	.5621	.5781	.5937	.6083	.6238	.6370	.6522	.6645	.6799	.6931	.7061
1.9	.5136	.5347	.5543	.5733	.5921	.6101	.6273	.6443	.6607	.6771	.6929	.7080	.7230	.7380	.7524	.7666
2.0	.5509	.5732	.5945	.6151	.6352	.6544	.6734	.6915	.7101	.7265	.7437	.7600	.7762	.7922	.8076	.8229
2.25	.6347	.6604	.6850	.7090	.7320	.7545	.7762	.7974	.8178	.8379	.8576	.8767	.8956	.9137	.9319	.9495
2.50	.7069	.7358	.7634	.7902	.8163	.8414	.8658	.8892	.9134	.9348	.9567	.9782	.9989	1.0196	1.0400	1.0597
3.0	.8277	.8615	.8943	.9259	.9563	.9860	1.0147	1.0427	1.0696	1.0962	1.1225	1.1477	1.1723	1.1966	1.2201	1.2438
3.5	.9252	.9631	1.0000	1.0352	1.0696	1.1032	1.1354	1.1665	1.1971	1.2271	1.2562	1.2846	1.3127	1.3400	1.3665	1.3925
4.0	1.0053	1.0470	1.0872	1.1265	1.1639	1.2005	1.2359	1.2701	1.3049	1.3363	1.3679	1.3992	1.4295	1.4593	1.4888	1.5171
5.0	1.1337	1.1814	1.2269	1.2710	1.3142	1.3555	1.3962	1.4346	1.4746	1.5099	1.5446	1.5814	1.6202	1.6503	1.6844	1.7166
6.0	1.2322	1.2849	1.3346	1.3833	1.4303	1.4758	1.5200	1.5625	1.6059	1.6450	1.6850	1.7233	1.7615	1.7987	1.8348	1.8699
7.0	1.3119	1.3681	1.4212	1.4734	1.5238	1.5725	1.6201	1.6673	1.7121	1.7536	1.7972	1.8375	1.8789	1.9196	1.9582	1.9958
8.0	1.3779	1.4376	1.4953	1.5488	1.6017	1.6539	1.7034	1.7519	1.8005	1.8452	1.8901	1.9338	1.9767	2.0192	2.0605	2.1002
9.0	1.4341	1.4971	1.5500	1.6135	1.6682	1.7222	1.7745	1.8252	1.8760	1.9227	1.9683	2.0154	2.0605	2.1044	2.1477	2.1891
10.0	1.4825	1.5478	1.6079	1.6684	1.7254	1.7818	1.8362	1.8880	1.9414	1.9892	2.0389	2.0859	2.1327	2.1794	2.2243	2.2672

TABLE XII - CORRECTION FACTOR FOR CHANGE IN GAS CONSTANT

Fuel-air ratio	Hydrogen-carbon ratio						
	0.084	0.100	0.125	0.150	0.175	0.189	2.00
	Correction factor, K_R						
0.01	0.9956	0.9966	0.9980	0.9994	1.0007	1.0014	1.0020
.02	.9914	.9932	.9961	.9988	1.0014	1.0028	1.0039
.03	.9871	.9899	.9941	.9982	1.0020	1.0041	1.0058
.04	.9830	.9867	.9923	.9976	1.0027	1.0055	1.0078
.05	.9789	.9835	.9904	.9970	1.0033	1.0068	1.0094
.06	.9749	.9804	.9886	.9965	1.0040	1.0081	1.0112
.07	.9710	.9773	.9868	.9983	1.0128	1.0206	1.0266
.08	.9790	.9908	1.0087	1.0258	1.0422	1.0511	1.0579
.09	1.0006	1.0139	1.0338	1.0529	1.0712	1.0811	1.0877
.10	1.0219	1.0365	1.0584	1.0794	1.0996	1.1105	1.1188
.11	1.0428	1.0587	1.0826	1.1055	1.1274	1.1393	1.1485
.12	1.0633	1.0805	1.1064	1.1311	1.1548	1.1677	1.1775

TABLE XIII - RATIO OF SPECIFIC HEATS OF COMBUSTION GASES

AT 1980° F ABSOLUTE

Fuel-air ratio	Hydrogen-carbon ratio						
	0.084	0.100	0.125	0.150	0.175	0.189	0.200
	Ratio of specific heats						
0.01	1.3222	1.3221	1.3220	1.3218	1.3217	1.3216	1.3216
.02	1.3158	1.3156	1.3153	1.3151	1.3149	1.3147	1.3146
.03	1.3096	1.3094	1.3091	1.3087	1.3048	1.3082	1.3081
.04	1.3038	1.3035	1.3031	1.3027	1.3024	1.3022	1.3020
.05	1.2982	1.2979	1.2975	1.2970	1.2966	1.2964	1.2963
.06	1.2929	1.2926	1.2921	1.2917	1.2913	1.2910	1.2909
.07	1.2879	1.2875	1.2870	1.2880	1.2900	1.2910	1.2917
.08	1.2873	1.2901	1.2925	1.2945	1.2964	1.2973	1.2980
.09	1.2947	1.2962	1.2985	1.3004	1.3021	1.3030	1.3036
.10	1.3005	1.3024	1.3040	1.3058	1.3073	1.3020	1.3080
.11	1.3059	1.3079	1.3091	1.3106	1.3119	1.3126	1.3131
.12	1.3110	1.3121	1.3137	1.3151	1.3162	1.3168	1.3172

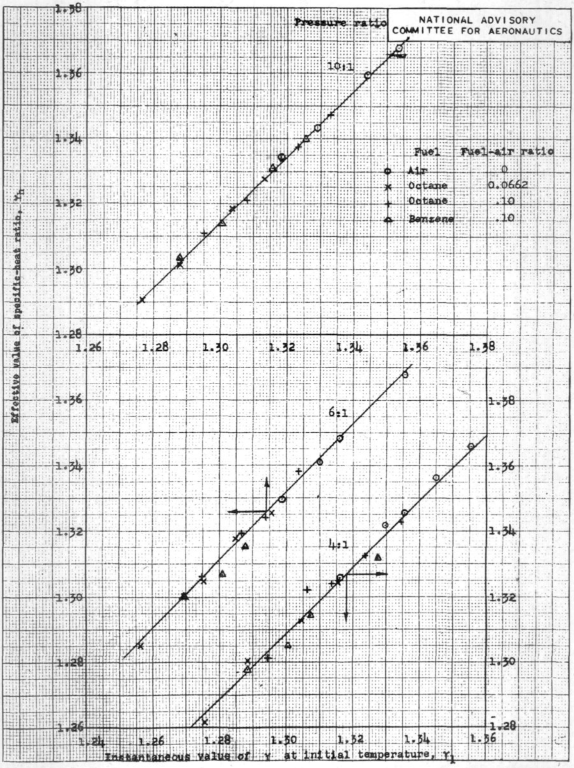

Figure 1. - Relation between effective and instantaneous values of γ for exhaust gas of various compositions.

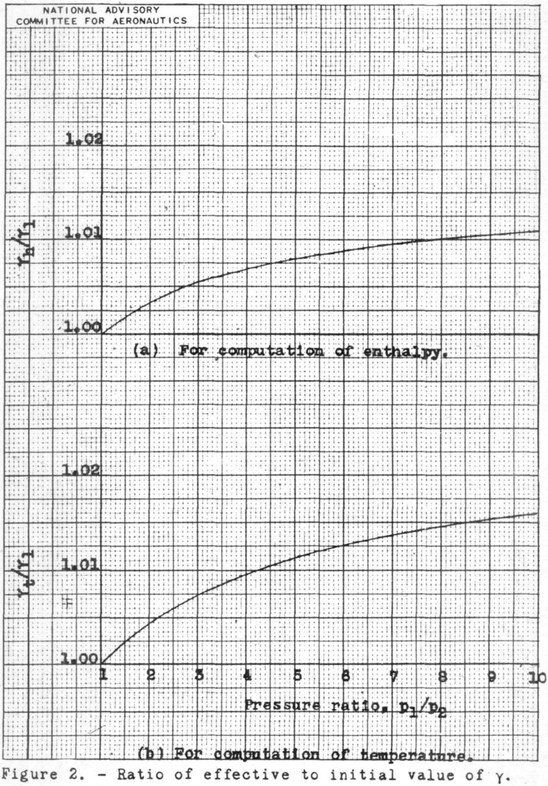

Figure 2. - Ratio of effective to initial value of γ.

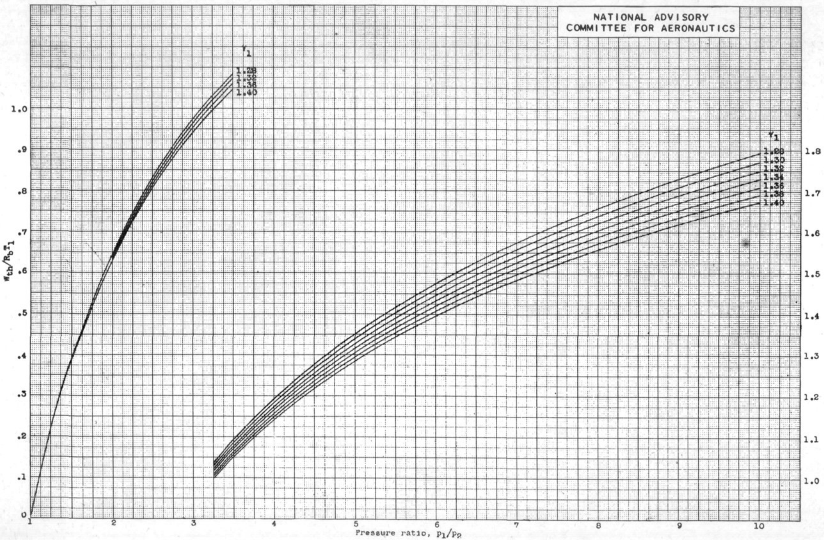

Figure 3. - Factor for computing work in an isentropic flow process. (An 11-in. by 17-in. print of this chart is attached.)

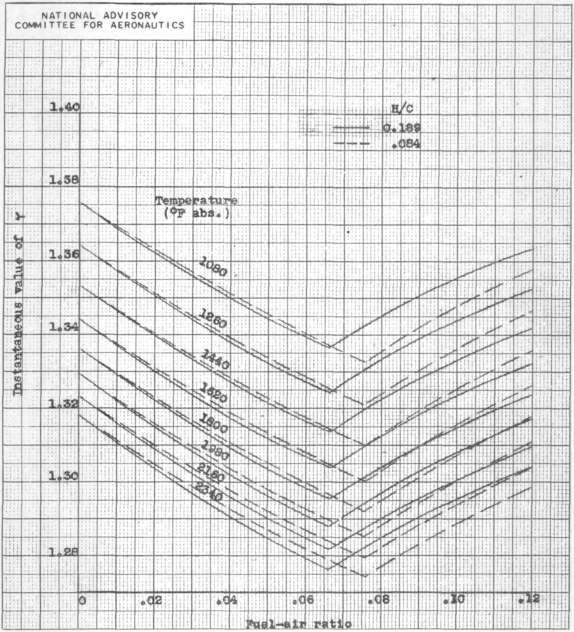

Figure 4. - Instantaneous values of specific-heat ratio γ for exhaust gas of various temperatures and compositions.

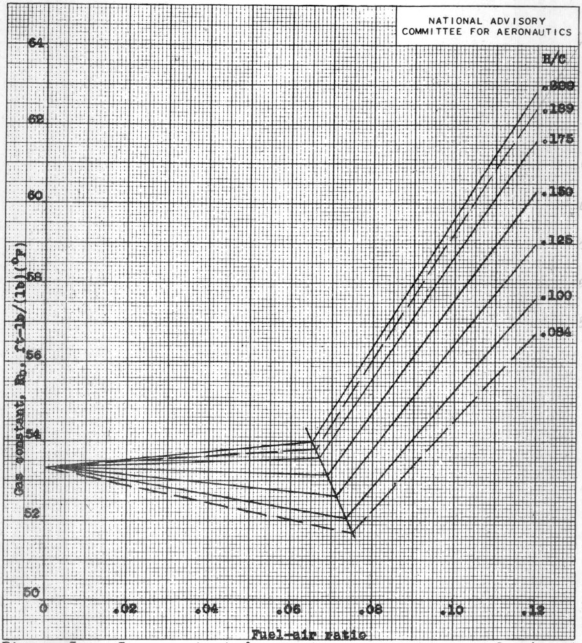

Figure 5. - Gas constant for various compositions of exhaust gas.

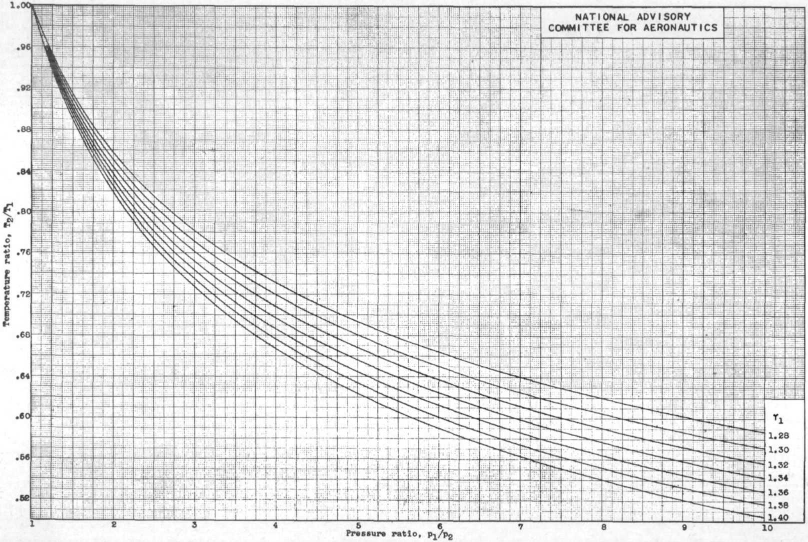

Figure 6. - Temperature ratio in isentropic expansion. (An 11-in. by 17-in. print of this chart is attached.)

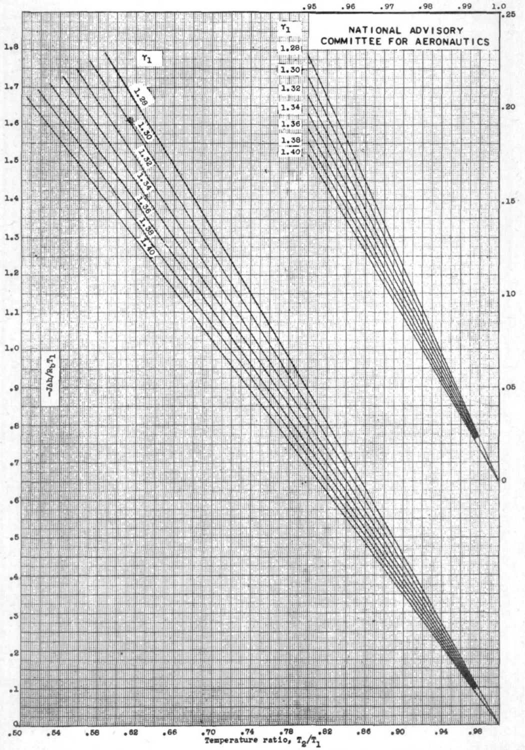

Figure 7. - Factor for computing change in enthalpy. (An 11-in. by 17-in. print of this chart is attached.)

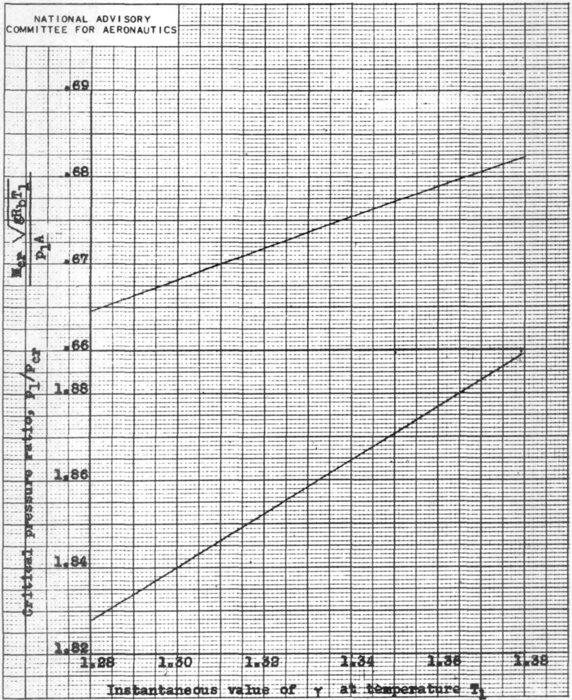

Figure 8. - Chart for determining critical mass flow and critical-pressure ratio.

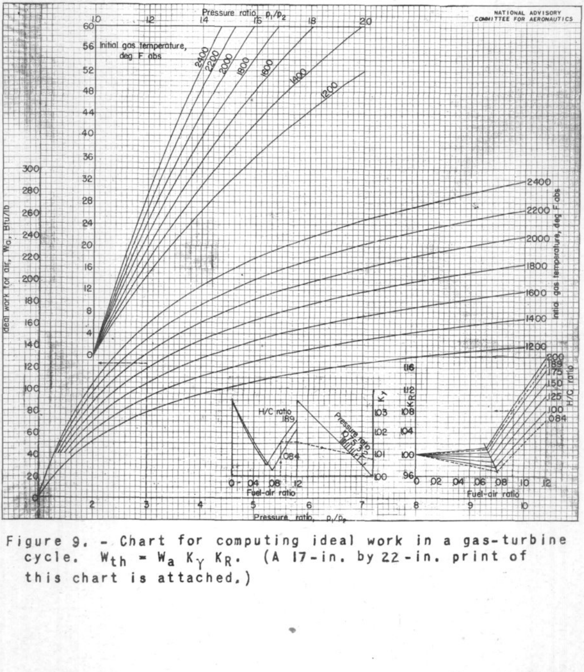

Figure 9. - Chart for computing ideal work in a gas-turbine cycle. $W_{th} = W_a K_\gamma K_R$. (A 17-in. by 22-in. print of this chart is attached.)

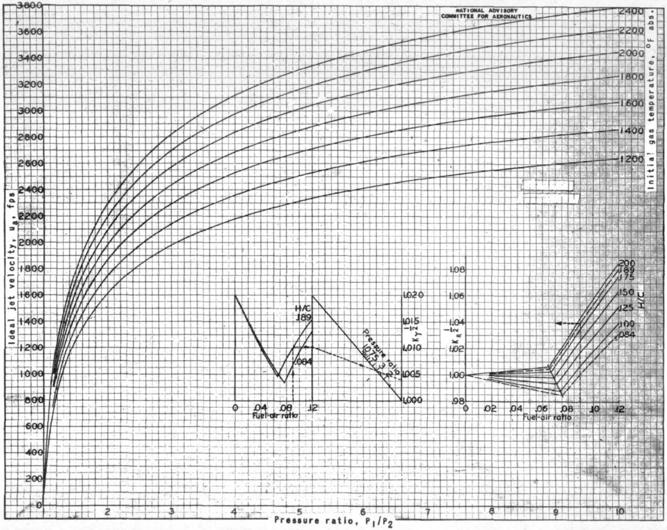

Figure 10. - Chart for computing ideal jet velocity. $u_b = u_a K_Y^{\frac{1}{2}} K_R^{\frac{1}{2}}$. (A 17-in. by 22-in. print of this chart is attached.)

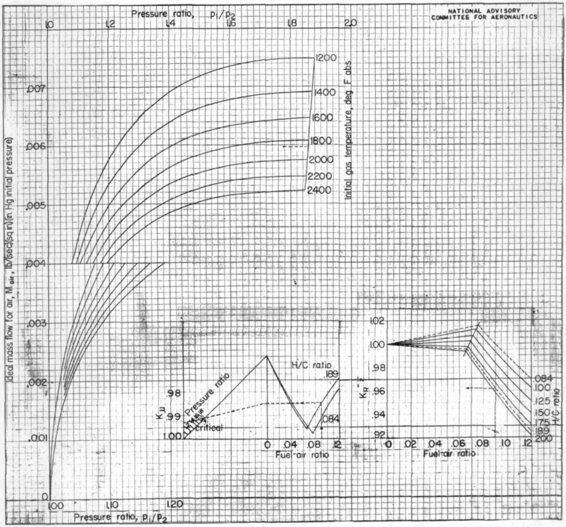

Figure 11. - Chart for computing ideal mass flow for convergent nozzle. $M_b = M_{air} K_R^{\frac{1}{2}} K_\mu$. (An 17-in. by 23-in. print of this chart is attached.)

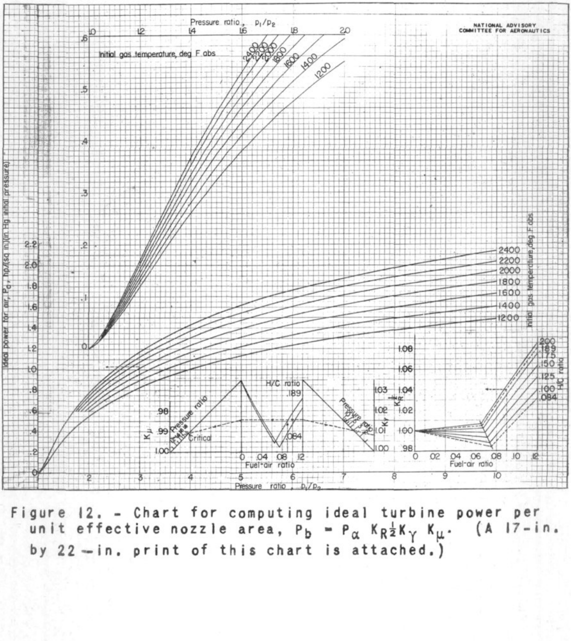

Figure 12. - Chart for computing ideal turbine power per unit effective nozzle area, $P_b = P_\alpha K_R^{\frac{1}{2}} K_\gamma K_\mu$. (A 17-in. by 22-in. print of this chart is attached.)

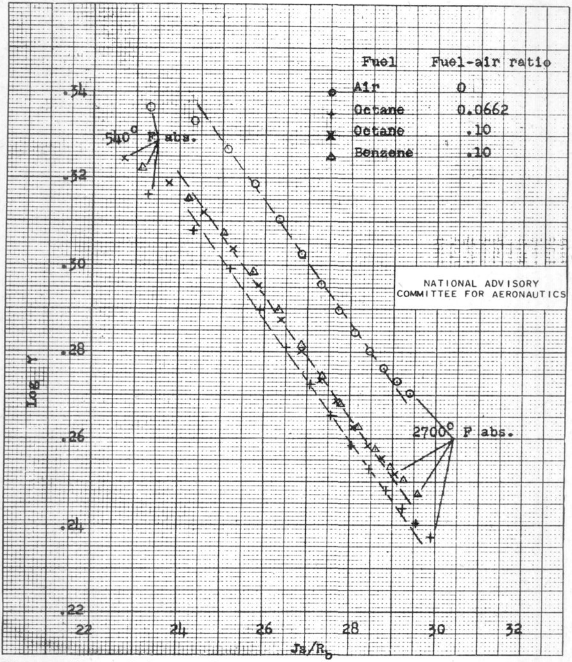

Figure 13. - Relation between logarithm of γ and entropy at 1 atmosphere pressure for combustion gases; temperature interval between points, 180° F.

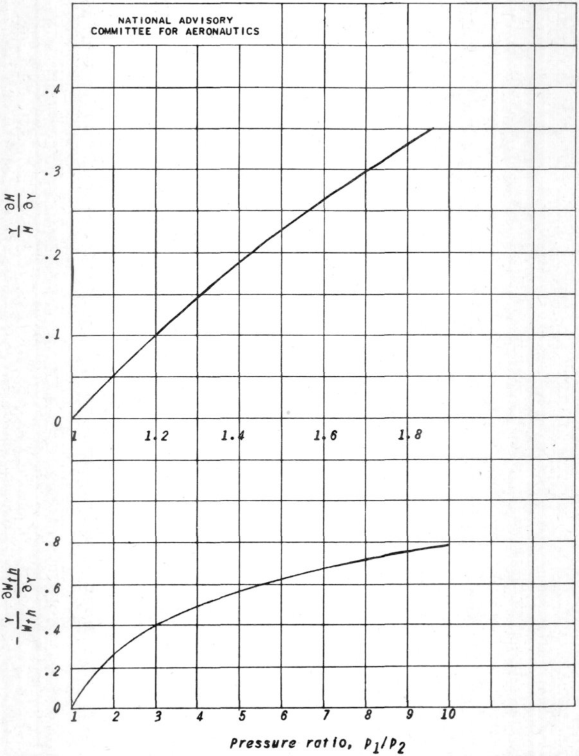

Figure 14. - Rate of change of available energy and ideal mass flow with changes in the ratio of specific heats.