Gearing for Gearheads

Part 2

by Phillip Miller

The geometry, tooth forms, ratios and dimensions of production as well as modified gearsets for the Rolls-Royce Merlin propeller speed reduction unit (PSRU) were presented in Part 1. We'll now explore the basic features and relations of involute spur gearing. These can be found in textbooks and references but not all in one place or in generally convenient forms. Many textbook presentations are aimed at accommodating the design of a gear train using standard momentarily popular tooth forms. They accordingly often contain constants and assumptions that prohibit their direct use in "reverse engineering" a gearset like the R-R Merlin's, which was at the cutting edge of technology when it was designed. We hope the results will be useful to AEHS Gearheads beyond study of the R-R Merlin PSRU.

The Involute Revisited

The involute of a circle (involute curve) is defined as the trace generated when the free end of a filament, held taut so as to form a continuously extending tangent, is unwound from a base circle. The involute curve has only one essential form but its size or scale varies with the size of the base circle. The pressure angle varies with position on the involute curve. A 25° pressure angle, for instance, is "farther out" than a 20° one (fig. 1).

A string wound tightly around a jar lid can be used as an involute generator on the drawing board. The lid is held down against paper and a pencil inserted through a loop in the string free end. When the string is unwound under constant tension, the pencil draws the involute curve. The jar lid can be traced to record the base circle.

Repeating this procedure with a variety of jar lids yields illustrations of different "scale" involute curves and is excellent for education and familiarization. The truths and utility of the involute curve in gear applications become clearer as they are actually generated in working combinations, not just studied in class or textbook.

Immediately it is seen that the base circle tangent filament, equal in length to the arc from which it is unwound, is the local involute radius of curvature and its point of tangency to the base

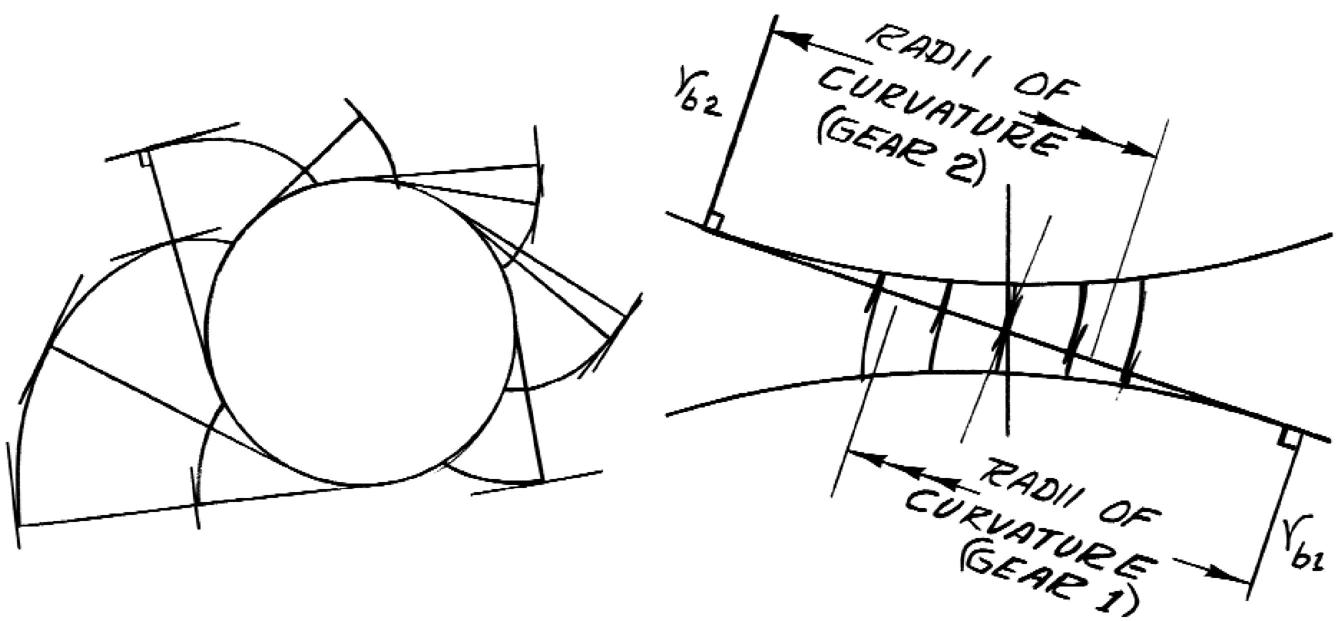


Fig. 1. (Left)An involute curve is formed at the free end, as a filament, held tightly as a tangent, is unwound from a base circle. The tangent is the local radius of curvature of the involute. Normals to the tangent ends are tangent to the involute. Any same-handed tangent to the base circle replicates these conditions when crossing an involute.
(Right) Line of Contact = Diagonal Mutual Tangent between Base Circles. Involutes of these circles meet in tangency along the contact line with normals to the contact line. Radii of curvature of these involutes, at the contact line, all lie along the contact line with their origins at the contact line origins.

circle is the "fixed" end of that local radius of curvature. It is obvious that any tangent to the involute curve at a filament's intersection or generating point on the curve is perpendicular to the filament/radius of curvature. This is a familiar scene with machine elements as it is also true of roller or ball bearings, cams against followers, etc. Any element or component in contact with the involute is, by definition, tangent to it .

Finally the realization dawns that all points of the involute curve are drawn by ends of tangents unwound from the base circle and hence ANY tangent to the base circle intersecting the involute is identical to and duplicates the generating tangent filament of that point. The resulting gifts of the involute to Gearheads are so profound that one wonders why they have not been seized upon by theologians. To wit, draw two circles in near proximity, and then lay a mutual tangent diagonally between them. Connect the circles' centers with a straight line crossing the diagonal mutual tangent and through this intersection place a short straight line perpendicular to the diagonal mutual tangent. This is the basic geometry of all mating involute spur gears. Involutes swung from each of the now recognized base circles through the mutual tangent and centerline intersection obviously share the perpendicular as a mutual tangent while the local radii of curvature lie in opposite directions along the diagonal tangent.

A further shock to the imagination waits: At any point on the diagonal mutual tangent, draw a short line crossing and perpendicular to the mutual tangent. Now swing an involute from each of the base circle mutual tangent intersections through that point. These share THAT short perpendicular as a tangent and this phenomenon is continuous all along the diagonal mutual tangent.

Gearing isn't composed of stationary base circles sprouting an infinite number of mating involutes but this view is well simulated by two rotating and meshing gears. The diagonal mutual tangent from base circle to base circle is termed the "contact line" (also "pressure line"). The involute teeth of a rotating meshing gearset contact one another only along this line (This remains true as long as the driving/driven gear selection and directions of rotation remain unchanged. Changing the direction of rotation creates a reversed or mirror image of the same basic scheme). The involute teeth remain in contact along the contact line and all contact positions along this line support uniform driven rotation with uniform driving rotation at the elementary gear ratio of the gearset. This is known as "The Conjugate Action of Involute Gears".

An observation that bears repeating is "there is only one involute curve!" It scales with its base circle and "standard" gearsets of 20° or 25° pressure angles merely make use of different portions of the involute curve. The base circle of a given pitch diameter 25° pressure angle gear is smaller than the base circle for a 20° pressure angle gear of the same pitch diameter. Thus 25° pressure angle gears make use of a closer to the "toe" portion of the involute's pointy-toed-cowboy-boot-like contour.

An oft proclaimed virtue of involute gearing is its relative immunity to changes in center-to-center distance. This can be seen in the left illustration of figure 2 (or *Torque Meter* Vol. 5, No. 1, pg 40) where changing the center to center distance of the two original base circles (as from a 20° to a 25° pressure angle) does not disrupt the existing geometry. The diagonal mutual tangent rotates a bit, the pressure angle increases uniformly in all its appearances and the resulting smaller value of the cosine of the now larger pressure angle operating on the increased implied pitch radii justifies, again, the original base circle radii.

That's fine but don't abuse the privilege! Increasing the center-to-center distance also increases the tooth bending stress, increases the gear separating loads and decreases the number of teeth in simultaneous contact (contact ratio). This can end up in a "bootstrapping" operation with increased noise and massive gearbox failure. Old folks who long ago ran increased displacement and highly provoked flathead Ford V-8s in front of post '39 "Sideloader" gearboxes are likely to retain such memories. This type of problem is not yet an endangered species.

While not directly involved, I have recently (well fairly recently) been sideswiped by the failed attempt by a large agricultural machinery manufacturer to use a "Sideloader" gearbox and also felt the disappointment of a wanna-be aircraft engine manufacturer who insisted on a long spur gear train in a too-light alloy housing. Steel or other high modulus plates centerline inset in light alloy housings to support and constrain gear shafts or bearings have long been a traditional cure for such problems.

The contemporary thought is that the near linear relation of material density to modulus invalidates the traditional solution as the weight increase will be the same whether a similar amount of stiffness is added by higher density steel inserts or just overall increased thickness of the light alloy gearbox. This misses the point. Centerline steel (or beryllium!) takes gear loads in nearly direct tension and compression. The walls of light alloy housings, even though thickened, receive these loads mainly through shear paths. This considerably reduces their overall center to center stiffness.

Contact Ratio

The contact line length (see fig. 2) determines the theoretical maximum number of teeth that may be in simultaneous contact (contact ratio) with a straight cut gearset of given pitch for the gear diameters in question. This number is typically not reached in practice and there are good reasons to limit the number of teeth in simultaneous contact. Achieving the highest possible contact ratio value requires use of the full tooth depth, i.e., progressive contact from a large outer diameter to a small root diameter. This full use of the tooth face has two obvious and immediate effects:

1) Sliding typically increases with increased contact departure from the "neutral point" and this CAN present wear problems at the high pitch line velocities found in high speed gearing.

2) The local involute radius of curvature decreases sharply as the base diameter is approached so contact pressure and Hertzian stress rise (more about this later) and can become a problem as contact extends toward the base circle. Early wear and surface failure in high power transmission service are potential results.

Accordingly, diametral pitches (i.e., tooth sizes) are typically chosen so that no more than 1.4 to 1.6 teeth are in contact at any one time. This is not "cast in concrete". For instance, light loading and the need for ultra quiet operation may well dictate a higher contact ratio.

The reasonable assumption that there exists sufficient involute tooth profile "below" the tangent pitch circles of a gearset (think root diameter/dedendum thoughts) to allow rotation with the intended tooth crown ODs (think addendum thoughts) allows a simple and accurate calculation of the effective contact line length and hence the actual contact ratio of the gearset (see fig. 2).

This total effective contact length divided by the normal pitch yields the contact ratio of the gearset. The normal pitch (tooth pitch along the contact line) is obtained by dividing the base circle circumference by the number of teeth in the gear.

Tooth Breakage

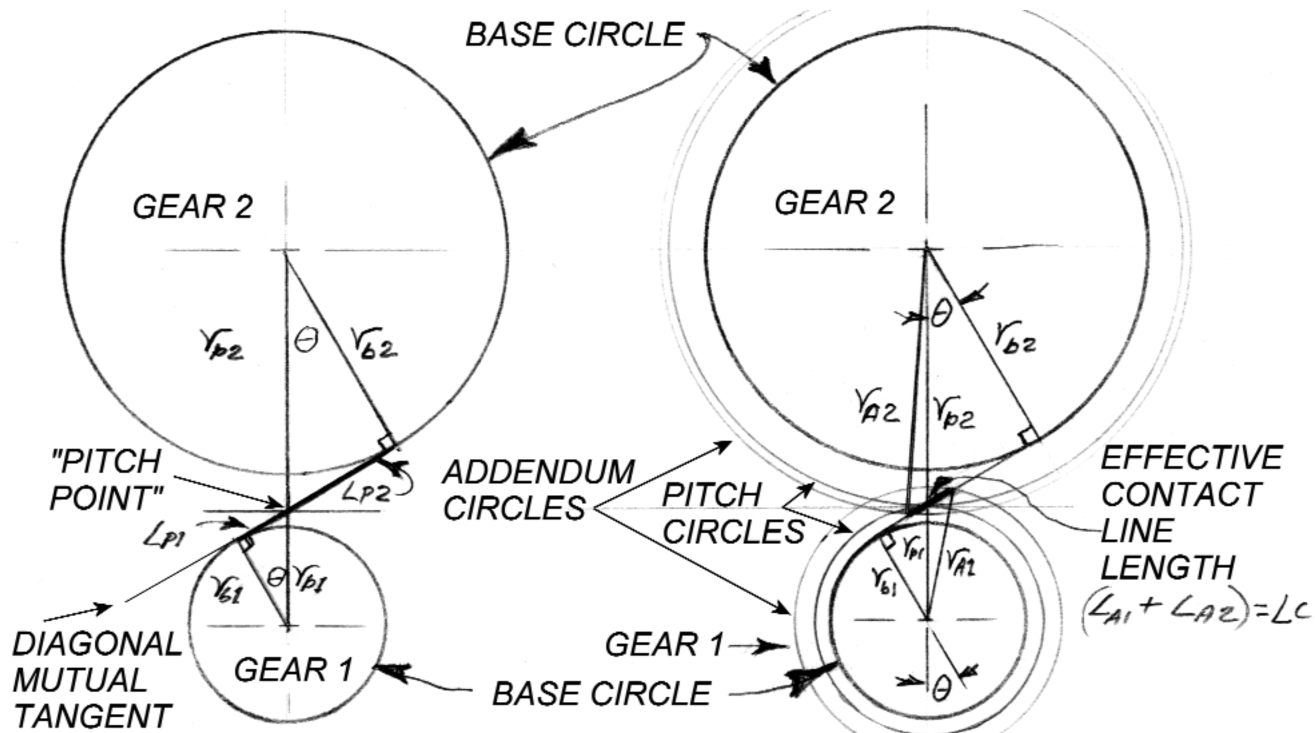
A primary concern for any gearset is tooth breakage in service. Merlin tooth breakage is a particular worry today as unlimited racers extend the service envelope, vary ratios and hence increase pitch line loads. Tooth breakage is not the simple concern it first appears. A loaded gear tooth resembles some sort of a beam in bending but this view is complicated by load sharing among multiple teeth, the rate of load application at speed (suddenly applied loads can double nominal deflection and stress), stress concentration at fillet radii and the fact that the stress equations don't apply without error to a short, wide, stubby and tapered beam even if the point of load application were to be essentially fixed by restricting the contact ratio.

Experience shows that a satisfactory gearing service life can be roughly predicted by calculation. A reasonable but all-in-all arbitrarily defined and located stress level and subsequent comparison to the material properties of the critical gear are involved.

In order to evaluate the tooth breakage potential of R-R PSRU gearing we begin by computing the maximum contact line load at the corresponding input rpm with the chosen ratio and pitch radii. The contact line load is equal to 33,000 x hp divided by the pitch line velocity (V) in feet per minute (where V = π x pinion base circle diameter x rpm/12). This load is hypothetically applied through a pinion tooth corner as the corner crosses the contact line. It is resolved at the tooth centerline (CL) intersection into radial and normal to the tooth CL components. This is illustrated and the geometry explored in figure 3.

The radial or tooth CL parallel component is applied along the tooth CL and is assumed to cause uniform compression loading in the region of the tooth base where it is generally ignored. This may not be entirely justified as will be seen later.

Text continued on page 37



TOTAL PRESSURE LINE

$$\frac{L_{P1}}{r_{b1}} = \tan\theta, \frac{L_{P2}}{r_{b2}} = \tan\theta$$

$$L_{P1} + L_{P2} = \tan\theta\left(r_{b1} + r_{b2}\right)$$

$$L_{P1} + L_{P2} = L_T$$

L_T (total pressure line length) will be used in the calculation of sliding velocities on the involute tooth surfaces over the effective contact line length

$$\frac{N_P}{N_G} = \frac{W_G}{W_P} : \frac{r_{P1}}{r_{P2}} : \frac{r_{b1}}{r_{b2}} : \frac{L_{P1}}{L_{P2}} = RA$$

where "*RA*" is the reduction ratio

EFFECTIVE CONTACT LINE

(determined by addendum radii intersections on pressure line)

$$\left(r_{A1}^2 - r_{B1}^2\right)^{1/2} = L_{P1} + L_{A1}$$

$$\left(r_{A2}^2 - r_{B2}^2\right)^{1/2} = L_{P2} + L_{A2}$$

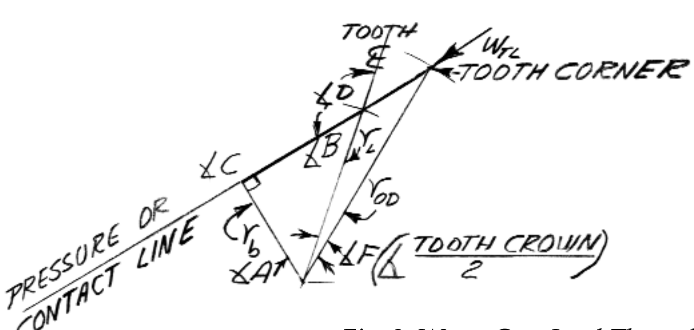
$$L_{P1} + L_{A1} + L_{P2} + L_{A2} - \left(L_{P1} + L_{P2}\right) = L_C$$

L_C is the effective contact line length

$$\frac{L_C}{NP} = CONTACT\ RATIO$$

CONTACT RATIO: number of teeth in simultaneous contact. ***NP*** (normal pitch) equals the base circle circumference ($2\pi r_b$) divided by the number of teeth in the gear $NP = 2\pi r_b / N$

Fig.2. Total Pressure Line and Effective Contact Line.
A right triangle is formed in one of the gears by the base circle radius, its right angle attachment to the contact line and the pitch circle radius of that gear coincident with the center-to-center line. The two radii are known. With help from Pythagoras we find the length of the contact line tangent from base circle to the gearset center-to-center line. Replacing the pitch circle radius with the (outer diameter) tooth crown radius so that its origin remains at the gear center and the other end intersects the contact line tangent on the far side of the gearset center- to-center line, Pythagoras once again helps find this newly defined length along the contact line. We then subtract from this newly defined line the previously found length of contact line from base circle to the center-to-center line to obtain the remainder as the portion of the effective contact line attributable to the OD radius/tooth crown radius of the gear. Repeating these efforts for the mating gear and adding the two effective contact line portions, we obtain the total effective contact length of the gearset.



$$\angle A = \cos^{-1}\frac{r_b}{r_{OD}} \quad , \quad \angle B = 90^o - \left(\angle A - \angle F\right)$$

$$\angle D = 180° - \angle B \quad , \quad r_L = \frac{r_b}{\sin\angle B}$$

$\angle B$ is the angle of load application at r_L at the tooth ℄ in the corner loaded condition.

Fig. 3. Worst Case Load Through a Single Tooth Corner

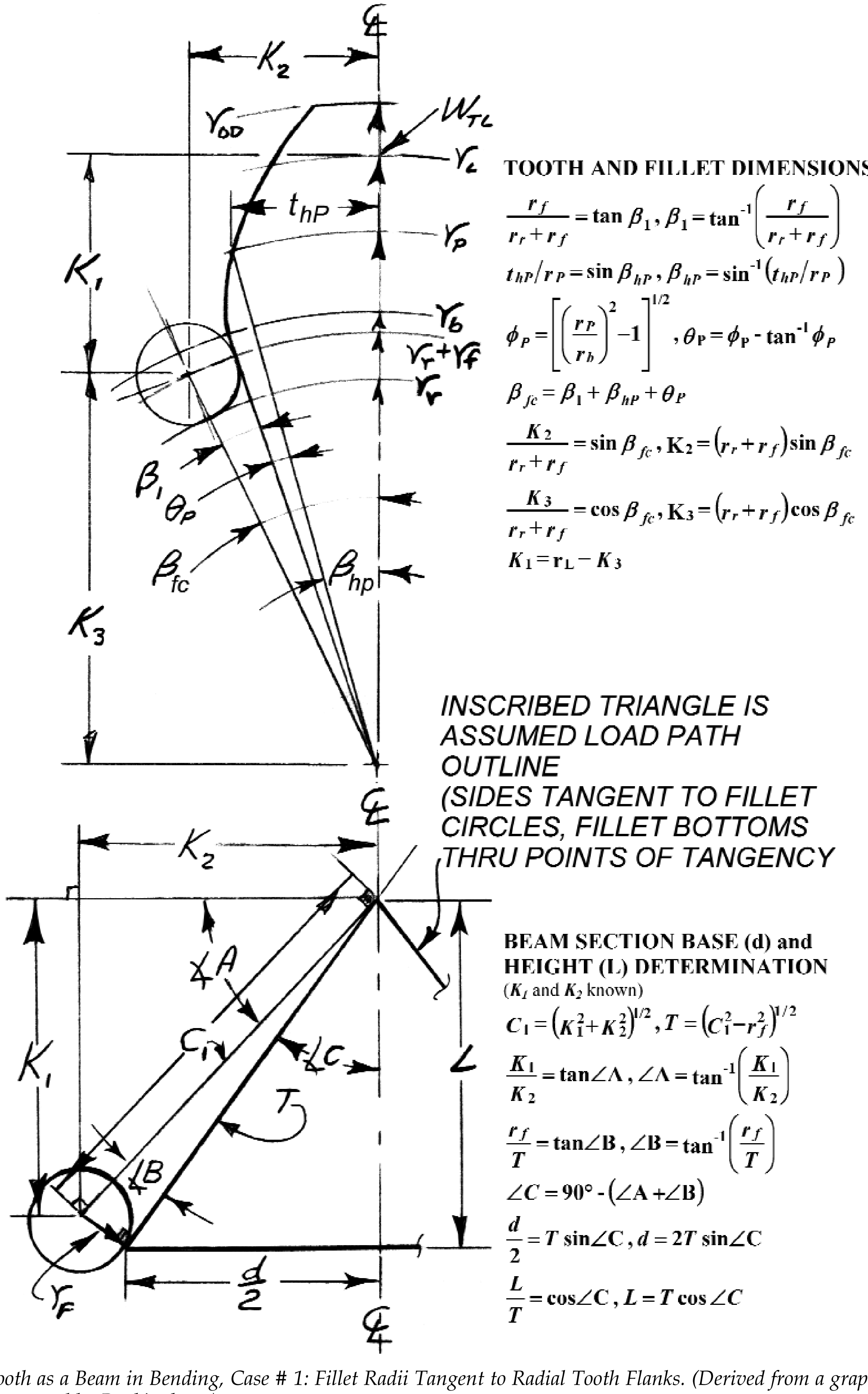
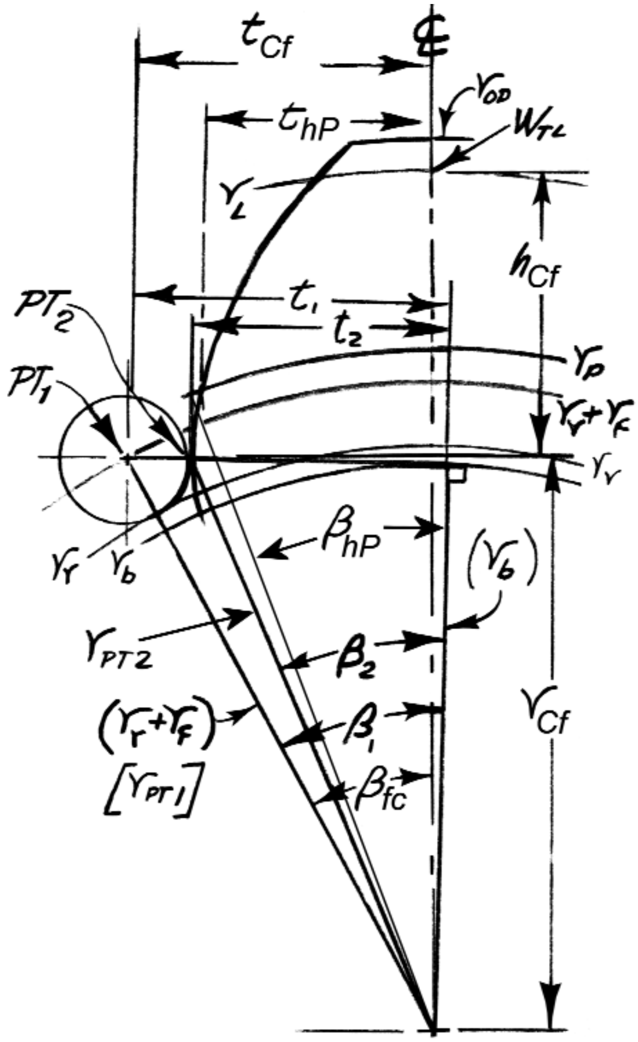
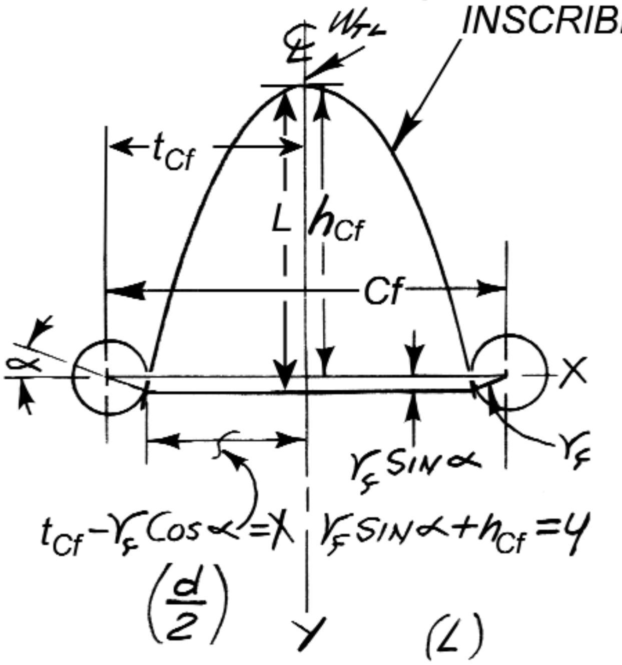


Fig. 4. Tooth as a Beam in Bending, Case # 1: Fillet Radii Tangent to Radial Tooth Flanks. (Derived from a graphical method presented by Buckingham.)





TOOTH AND FILLET DIMENSIONS

$$t_1 = \left[\left(r_r + r_f\right)^2 - r_b^2\right]^{1/2}, \quad t_2 = t_1 - r_f$$

$$r_{PT2} = \left(t_2^2 + r_b^2\right)^{1/2} \therefore \; r_{PT2} = \left[\left(t_1 - r_f\right)^2 + r_b^2\right]^{1/2}$$

ANGULAR CONTRIBUTIONS

$$\phi_2 = \left[\left(\frac{r_{PT2}}{r_b}\right)^2 - 1\right]^{1/2}, \quad \theta_2 = \phi_2 - \tan^{-1}\phi_2$$

$$\phi_P = \left[\left(\frac{r_f}{r_b}\right)^2 - 1\right]^{1/2}, \quad \theta_P = \phi_P - \tan^{-1}\phi_P$$

$$t_{hP}/r_P = \sin\beta_{hP}, \quad \angle\beta_{hP} = \sin^{-1}\left(t_{hP}/r_P\right)$$

$$\frac{t_1}{r_b} = \tan\beta_1, \quad \beta_1 = \tan^{-1}\left(\frac{t_1}{r_b}\right)$$

$$\frac{t_2}{r_b} = \tan\beta_2, \quad \beta_2 = \tan^{-1}\left(\frac{t_2}{r_b}\right)$$

$$\beta_1 - \beta_2 = \beta_{f(r)}$$

$$\beta_{fc} = \beta_{f(r)} + \beta_{hP} + \theta_P - \theta_2$$

β_{fc} is the angular width from fillet ℄ to tooth ℄ (half tooth and fillet radius sum)

$$t_{Cf}/\left(r_r + r_f\right) = \sin\beta_{fc}, \quad t_{Cf} = \left(r_r + r_f\right)\sin\beta_{fc}$$

t_{Cf} is half the fillet chordal distance

$$\frac{r_{Cf}}{\left(r_r + r_f\right)} = \cos\beta_{fc}, \quad r_{Cf} = \left(r_r + r_f\right)\cos\beta_{fc}$$

$$h_{Cf} = r_L - r_{Cf}$$

h_{Cf} is the fillet-to-filet chord radial location

BEAM SECTION BASE (d) and HEIGHT (L) DETERMINATION from h_{Cf} and t_{Cf}

$$x^2 = a\,y, \quad x = \pm\sqrt{a\,y}, \quad 2x\,dx = 2\,dy$$

$$m = \frac{dx}{dy} = \tan\alpha = \frac{a}{2x}$$, but from $x^2 = a\,y$,

$$a = \frac{x^2}{y} \;\therefore\; \frac{dx}{dy} = \frac{x^2}{2x\,y} = \frac{x}{2\,y}$$

$$t_{Cf} - r_f\cos\alpha = x, \quad \text{and } x = \frac{d}{2}$$

$$r_f\sin\alpha + h_{Cf} = y, \quad \text{and } y = L$$

substitution yields:

$$\frac{t_{Cf} - r_f\cos\alpha}{2\,h_{Cf} + 2\,r_f\sin\alpha} = \tan\alpha$$

Tan *α* of filet circle matches slope ***m*** of the inscribed parabola at points of tangency. Solve by iteration for ***α*** and evaluate to find beam section dimensions ***d*** and ***L***.

Fig. 5. Tooth as a Beam in Bending, Case #2: Fillet Radii Tangent to Involute Tooth Flanks. (Derived as a precursor to analytic determination of tooth beam dimensions, equivalent to a graphical method presented by Spotts.)

Gearing for Gearheads
Text continued from Page 33

The CL normal component is the bending load for our short, wide, stubby, and tapered tooth beam. It is applied at the tooth CL/tooth corner load intersection.

The tooth (beam) critical section in bending and its location are determined by one of two arbitrary but similar techniques, one for the 65-tooth gearset and one for the 71-tooth gearset, both of which were described in Part 1.

Case #1: Triangular Load Path

Case #1 occurs when the gear tooth shape is a bit re-entrant because the root circle is sufficiently smaller than the base circle that the fillet radii are tangent to radial tooth flanks inboard of the involute surfaces. Fillet radii are, of course, also tangent to the root circle. A straight side triangular load path core is assumed within the tooth profile. The apex is at the intersection of tooth CL and contact line through a tooth corner. The triangle sides are tangent to the fillet circles and the triangle base is the line connecting these tangent points. The triangle base is taken as the beam critical section thickness in bending. Calculation of the critical section and effective beam length are presented in figure 4.

Case #2: Parabolic Load Path

Case #2 occurs when the base circle is smaller than the root circle. Fillet circles in this case are tangent to the involute gear flanks and the root circle. The tooth profile is entirely involute to the fillet tangencies and beyond. There is no convergence or re-entrance to this shape and an inscribed parabola with peak through the CL corner tooth load/contact line intersection and tangent to the fillet circle is an appropriate load path assumption. Again, beam depth or thickness is the straight line between the parabolic load path fillet tangencies. The beam length is the radial distance from this base line to the CL/parabola peak. Unfortunately, attempts to write a rational equation for this arrangement end up one notch worse than cubic. However iteration (a public relations term for trial and error) rolls out the solution to three places in a few steps. This is presented graphically and algebraically (without the iterative steps) in figure 5.

Evaluating Tooth Stresses

The tooth beam dimensions L and d have been calculated, they may be used with the tooth CL normal force value and the tooth face width (b) to obtain peak equal tension and compression stresses at the tooth critical section base. If this stress value is roughly HALF of the useful gear material strength and none of the values for pitch line speed, contact ratio, rubbing velocity, or dynamic load are out of line, there is a reasonable chance for success and cause to proceed with further analysis, fabrication and test.

Today, the use of standard gearing is made convenient by extensive tables of gear tooth form factors, also called Lewis Factor "Y" values that incorporate tooth beam L and d terms, are scaled by tooth size (circular pitch, or p) and tabulated

LEWIS EQUATION - Tooth as a beam in bending.
From a stress analyst's equation:

$$S = \frac{M\,c}{I}\ ,\ M = F_b \times L\ ,\ c = \frac{d}{2}\ ,\ I = \frac{b\ d^3}{12}$$

where ***d*** is thickness and ***b*** is width at the base of the assumed equivalent cantilever beam. ***I*** is the base section (area) moment of inertia. Bending moment (***M***) is the product of force (F_b) times the application to base lever arm (***L***) and ***c*** is the lever arm of the induced surface bending stress (***S***). We now find that:

$$F_b \times L \times \frac{d}{2} \times \frac{12}{b\ d^3} = S = \frac{F_b}{b} \times \frac{6L}{d^2}$$

Continuing in the pioneering footsteps of Mr. Wilfred Lewis of the Philadelphia Engineers Club (circa 1892):

Let $p\,Y = \frac{d_2}{6L}\ \therefore\ Y = \frac{d_2}{6L\,p}$

This allows us to write: $S = \frac{F_b}{b\,Y\,p}$

as the maximum bending stress (tension & compression) and to permit scaling "***Y***" values or "Lewis factors" by tooth size (i.e., ***p*** or circular pitch, where
p = π x pitch diameter / number of gear teeth) and listing or cataloging them by the number of teeth in a gear.

The force (F_b) can be derived from pitch line load (***hp @ rpm***) such that:

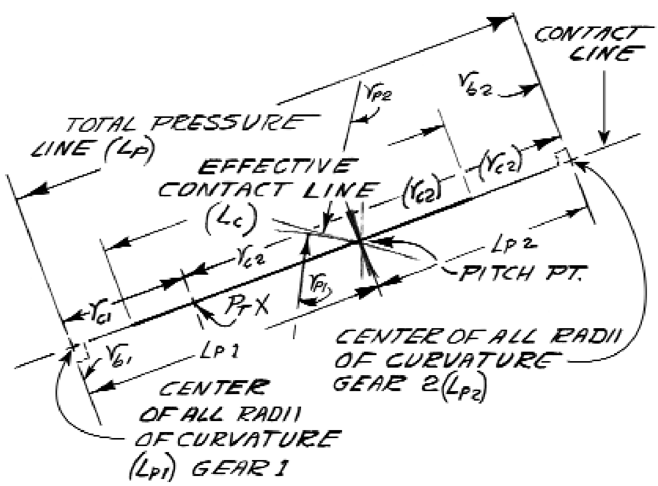
$$F_b = W_{TL} \times \cos(\angle D - 90°)$$

and $W_{TL} = \frac{198000 \times hp}{\pi \times r_b \times rpm}$

where r_b is the base circle radius.

Fig. 6. Derivation of the Lewis Equation for a Gear Tooth as a Beam in Bending.

by the number of teeth in a gear. This is quick and convenient but unfortunately does not help to reverse engineer gearing that was on the cutting edge long ago and for which specific tooth characteristics and resulting Lewis Factors did not find their way into our present Tables. However Lewis Factor "Y" values are easily derived ($Y = d^2/6Lp$) for comparison with today's standard tooth forms (fig. 6).



$$V = \frac{2\pi r_{p1} n}{12}$$ where V is pitch line velocity (fpm)

in terms of pinion pitch radius(r_{p1}) and rpm (n)

$$m\omega_1 = \frac{12V}{r_{p1}}, \quad m\omega_2 = \frac{12V}{r_{p2}}, \quad r_{C1} + r_{C2} = L_P$$

($m\omega$ is in radians per minute)

At any point X, $r_{C1} + r_{C2} = Lp$ and:

$$V_S = \frac{r_{C1}\omega_1 - r_{C2}\omega_2}{12}$$

V_S is the rub or slide velocity in fpm. This is often represented as $\frac{V_S}{V} = \frac{r_{C1}\omega_1 - r_{C2}\omega_2}{12V}$ (Note 1) and plotted above and below pitch line velocity at close intervals along the effective contact line length. The effective contact line length may also be calibrated in terms of radii to points on the involute teeth of either gear. (Note 2)

Note 1: Radii at point x:

$$r_{x1} = \left(r_{b1}^2 + r_{C1}^2\right)^{1/2} \;\&\; r_{x2} = \left(r_{b2}^2 + r_{C2}^2\right)^{1/2}$$

Note 2: Pure rolling contact ($V_S = 0$) always exists where

$$\frac{L_{P1}}{L_{P2}} = \frac{\omega_2}{\omega_1}, \quad L_{P2} \times \omega_2 = L_{P1} \times \omega_1 \therefore V_S = 0$$

(by geometric definition) at the "pitch point" (crossing of the gearset mutual center line and contact line.

Fig. 7. Gear Tooth Rub or Slide Velocity

Rolling and Rubbing

Teeth of equally-sized involute spur gears have pure rolling contact at the pitch point where the contact line crosses the gear center-to-center line. This point defines the radii of the two tangent circles (pitch circles), each of which is concentric to a base circle.

The radii of curvature of two mating teeth of two equal gears are equal at this point and the angular velocity of the two equal gears is also equal. The local velocity of a gear tooth surface is equal to the product of the local radius of curvature times the angular velocity of the gear. Remembering "equals times equals" from long ago, we see that we have a velocity match at this point and hence pure rolling contact.

Away from this point the radii of curvature of the mating teeth are unequal portions of the contact line. The products of angular velocity and radii of curvature are therefore unequal and there is a component of sliding or "cam type" contact equal to the difference between the two radii of curvature times angular velocity products. It is typically an oscillating or lapping action, with motion in one direction as the mating gears approach the neutral point and opposite motion as they leave it. Unequally sized gears (i.e., reduction gears) often show an effective contact line that is not centered about the neutral point and have higher sliding velocities.

An exercise with similar triangles as in figure 7 brings a pleasant surprise: pure rolling contact ALWAYS exists where the contact line crosses the mutual centerline of two meshed gears. In other words, the neutral point remains in place regardless of the gear ratio.

Sliding velocity (plus and minus) plotted with pitch line velocity as a "median" reference along the line of contact illustrates the phenomena well. Radial positions along a gear tooth correspond to positions on the line of contact and may be used as an alternate ordinate.

The physical effects of this sliding component of involute gear tooth contact are not often detrimental when the contact ratio is small and it then compares favorably with the ball/race sliding contact in a ball bearing. Careful examination of a good running ball bearing after substantial service reveals three continuous and distinct ball tracks typically predominant in the outer race. The central track is due to the ball equatorial diameter rolling against the race. The outboard two tracks are from each ball's smaller spherical segment diameters (on each side of the equatorial diameter) skidding against the race as the ball's rolling rpm is insufficient for these smaller diameters in their travel about the raceway.

Good running gear teeth usually pick up a burnished shine from their oscillatory sliding action, ball bearing raceway unidirectional sliding typically results in a slightly dull wear surface.

Dynamic Loads

Mean contact load on PSRU gearing is easily reckoned. However, the thumps, bumps and inertia coupling antics of a reciprocating internal combustion engine plus inevitable dimensional errors in gear manufacture can lead to repetitive loads much greater than mean values on PSRU gears trapped between the rotating engine and propeller masses. A shudder strong enough to just drop the load to zero between teeth of two gears in contact at their pitch radius (and hence just before the teeth carry the entire contact line load) will cause the load to effectively double when the gears resume contact. The effect is MUCH worse, of course, if the bump or shudder actually separates the two teeth and powered acceleration occurs across the closing gap.

Dynamic loading of this sort was investigated by an American Society of Mechanical Engineers committee circa 1931. The results are very rigorously reported by Earle Buckingham (a member of this committee) in his *Analytical Mechanics of Gears* and in a much more user friendly but less rigorous fashion in Merhyle Franklin Spotts' *Design of Machine Elements*, where two dynamic load equations are presented:

1) The "Barth" equation considers the nominal contact line load and a modifying factor involving only the pitch line velocity and so seems more suited to heavier and cruder industrial applications.

2) The "Buckingham" equation, in contrast, involves gear error tolerance, coupled masses, gear material resilience and coupling elasticity. It seems well suited for application to the R-R Merlin PSRU with its precise (0.0005" error presumed) hardened and ground gears, and with its impulse attenuating torsionally flexible quill shaft coupling the crankshaft and pinion. Further the Buckingham equation in its Spotts presentation is in a form which allows extraction of constant ("C") covering the net effect of the listed variables for cases where service experience permits prior determination of a sustainable dynamic load.

Gearsets subjected to excessive dynamic loading exhibit pitting, flaking and abrasive wear of tooth working surfaces and eventually fail. Fortunately, this failure mode may be caught BEFORE catastrophic structural damage and has in general proven to be predictable from examination of the dynamic load. Design parameters are chosen so that bending stress equals no more than one half the gear material tensile strength (this is nominally also the endurance limit in reversed bending). This, at first glance, suggests a "safety factor" of approximately two but covers less than half the story. Typical contact ratios place around 1.5 teeth in simultaneous contact near the pitch circle at all times, thereby dividing the contact line load between teeth. Also, the points of load application, on average, are well down from the highly-leveraged corner. This all serves to produce a gearset that will handle the nominal contact line load without tooth failure in bending. With reasonable attention to gear surface hardness, finish, lubrication and coupling flexibility between the engine and propeller rotating masses, it also provides a margin between nominal pitch line load and dynamic load that avoids surface pitting, flaking or pitting induced abrasive wear.

However, oscillation caused by reciprocating engine foibles and gear tooth error in a new and undeveloped or recently modified system can certainly surprise us with severe loads at a frequency that leads to the variety of surface fatigue failures noted above. Accordingly it is always wise to analytically investigate expected dynamic loading and materially sustainable Hertzian stress at the pitch radii of a gearset under study even though the corner loaded single tooth bending stress appears satisfactory.

That's All for Now

Part 3 will complete the tools necessary to compare gear stresses to material properties. These will be used to evaluate the potential service performance of the various stock and modified gearsets for the R-R Merlin PSRU in today's environment including unlimited racing.

Errata - Part 1 (*TM* Vol. 5, No. 1)

Page 39: pitch diameter = 5.5827, not 5.5872.
Page 40: base diameter = 5.24602, not 5.25025.

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